

Fomin's conception of quantum cosmogenesis

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The main aim of this paper is to extend the early approach to quantum cosmogenesis provided by Fomin. His approach was developed independently to the well-known Tryon description of the creation of the closed universe as a process of quantum fluctuation of vacuum. We apply the Fomin concept to derive the cosmological observables. We argue that Fomin's idea from his 1973 work, in contrast to Tryon's one has impact on the current Universe models and the proposed extension of his theory now can be tested by distant supernovae SNIa. Fomin's idea of the creation of the Universe is based on the intersection of two fundamental theories: general relativity and quantum field theory with the contemporary cosmological models with dark energy. As a result of comparison with contemporary approaches concerning dark energy, we found out that Fomin's idea appears in the context of the present acceleration of the Universe explanation: cosmological models with decaying vacuum. Contemporary it appears in the form of Ricci scalar dark energy connected with the holographic principle. We show also that the Fomin model admits the bounce instead of the initial singularity. We demonstrate that Fomin model of cosmogenesis can be falsified and using SNIa data the values of model parameters becomes in good agreement with observations.

I. INTRODUCTION

Cosmogony is defined as a study of the origin (cosmogenesis) of the Universe in the physical aspect. It asks the question how the Universe came into being. By comparing this process with human life, we called the birth of the Universe cosmogenesis [1, p.515]. Among different conceptions of cosmogenesis which offer the possibility of describing the origin of the Universe as a physical process, there are cosmogenic theories of the spontaneous creation from vacuum instability. According to these theories, the Universe was created in a spontaneous way, from a quantum fluctuation borrowed from the created Universe. Edward Tryon [2, 3] is usually considered as the scientist who first supported such theories in 1973 by arguing that total mechanical energy of the closed Universe is zero and effects of quantum fluctuation of the vacuum were important when the Universe was coming to the existence.

The second approach developed independently by Fomin [4, 5] in 1973[30] was based on the idea that the closed Universe originated from gravitational instability of the vacuum is realized in the background of general relativity theory, rather than on Newtonian one, like in Tryon's paper. Fomin's contribution is only addressed to in a footnote in Vilenkin's paper on quantum tunneling [6].

The main aim of this paper is to show that Fomin's idea offers deeper understanding of the origin of the Universe from the vacuum fluctuation for two reasons: the mechanism proposed by Fomin is based on Einstein general relativity in contrast to Tryon's idea formulated on the background of the Newtonian theory of gravity. Moreover, Tryon's idea cannot concern the curved closed universe. Nevertheless, the Newtonian theory can be useful in the context of discovery, but of course, it does not describe the early epoch of the universe when both gravitational and quantum effects plays a crucial role.

We can discover a new contemporary context for Fomin's theory by considering the cosmological models in which his idea is realized, providing the conservation condition is postulated. We find two basic categories of the cosmological models in which the universe is generally relativistic and originated from quantum fluctuation via Fomin's proposition. We also demonstrate that in contrast to Tryon's model, the models obtained in this paper can be confronted with modern cosmological observations of the distant supernovae SNIa. The critical analysis of Tryon's paper is not the subject of the present paper. Some interesting remarks and critiques of Tryon's idea can be found in McCabe's papers [7, 8]. We share McCabe's comments on Tryon's paper in many points but the correct definition of the energy seems to be crucial for the investigation of the creation of the Universe. Many authors have concluded recently that energy of the flat and closed FRW universes are equal to zero locally and globally. Unfortunately such conclusions originate from coordinate dependent calculations which are performed in the special comoving coordinates (called the Cartesian coordinates), by using energy momentum tensor of matter and gravity [9].

However, let us note that all expressions for the conserved energy-momentum tensor in general relativity can be written as a divergence of a superpotential. The integral over space can then be expressed as a surface integral over

the boundary, which is zero for the closed Universe. This conclusion does not depend on which metric or which coordinate system is used

The organization of this paper is as follows. In Section 2 we present Fomin's idea in details. Section 3 is devoted to the construction of cosmological models from basic Fomin's assumptions — first principles. In Section 4 these cosmological models are tested by the data from observation of distant supernovae type Ia, which offer the possibility of explaining the present acceleration of the Universe.

II. FOMIN'S IDEA OF THE ORIGIN OF THE UNIVERSE FROM QUANTUM FLUCTUATIONS

Fomin's main goal (like Tryon's) was to describe the origin of the Universe as a whole without violation of any conservation laws. Since at that moment gravitational fields were very strong, the gravitational interaction of the primordial vacuum with gravity should be taken into account. In both Tryon's and Fomin's papers, the vacuum is chosen as an initial state of the Universe (a metagalaxy in Fomin's terminology). Note that if strong gravitational fields are important, then effects of general relativity (not special relativity) should be included at the very beginning. This is the very reason why Fomin formulated the problem in the context of general relativity. In the introduction to his paper we can find an analogy to the Newtonian cosmology, but he noted the absence of the kinetic term ($\dot{a}^2/2$) in the energy balance equation $Mc^2 + V(M, a) = 0$ where M is the mass of the universe particle and $V(M, a)$ is the potential energy. It would be worth mentioning at this point that the notion of energy for the FRW models is well defined from the physical point of view because we are not dealing with isolated systems which are asymptotically flat spacetimes. Note that formally, from Pirani's or Komar's formula we can obtain zero energy for the Friedmann-Robertson-Walker (FRW) models but there is no time Killing vector and we have geodesic congruence of privileged observers. That is why, in our opinion, this result does not make much physical sense. Moreover, the result of energy calculations following Pirani's or Komar's formula depends on the choice of a coordinate system.

In the introduction to Fomin's paper we find remarks that the process of quantum cosmogenesis should be considered in the presence of a strong gravitational field and therefore the effects of general relativity should be included at the very beginning. In contrast to Tryon, Fomin considered an evolving non-static universe. It is well known that the FRW dynamics can be represented by the notion of a particle of unit mass moving in the potential, following an equation analogous to the Newtonian equation of motion, namely

$$\mathcal{H} = \frac{\dot{a}^2}{2} + V(a) \quad (1a)$$

$$\ddot{a} = -\frac{\partial V(a)}{\partial a} \quad (1b)$$

where $p_a = \dot{a}$ and a are generalized momenta and coordinates, respectively, H is the Hamiltonian and V is the potential function of the scale factor a , if V is a decreasing function of a , the Universe is accelerating [10, 11].

System (1) should be considered on the energy level $H = E = -\frac{K}{2}$ where $K = 0, \pm 1$ is the curvature constant (we use here and in next sections the system of units in which $8\pi G = c = 1$). Therefore, only for a flat relativistic system we have the energy level E equals to zero. It is interesting that the FRW model based on Fomin's conception can be formulated in terms of Hamiltonian systems of a Newtonian type in the form (1) with the corresponding function of the scale factor $V(a)$.

In Fomin's scenario, the Universe emerged from a physical state, which he called the "null vacuum" system S_V . Vacuum is assumed as an initial state of further evolution of the Universe. This system produced particles and antiparticles in a spontaneous manner without violating any conservation laws. Then emerging matter of mass m , (called bimatter by Fomin), gives rise to the growth of entropy. Fomin argued that such a process of decaying vacuum is energetically preferred and both entropy and the number of particles and antiparticles will be growing at the cosmological time. The growth of the number of particles is strictly related to the size of the system, as we assume that the created Universe is closed (see Fig. 1).

Fomin pointed out that for the description of the contribution of decaying vacuum (in a phenomenological way), the Einstein field equations should be generalized. Hence the vacuum is characterized by an energy momentum tensor of the form

$$T_{\mu\nu}^{\text{vacuum}} = -\lambda(x)g_{\mu\nu} \quad (2)$$

where the energy density of vacuum λ depends on the scalar curvature of the spacetime

$$\lambda(x) = k\kappa^{-1}R(x) \quad (3)$$

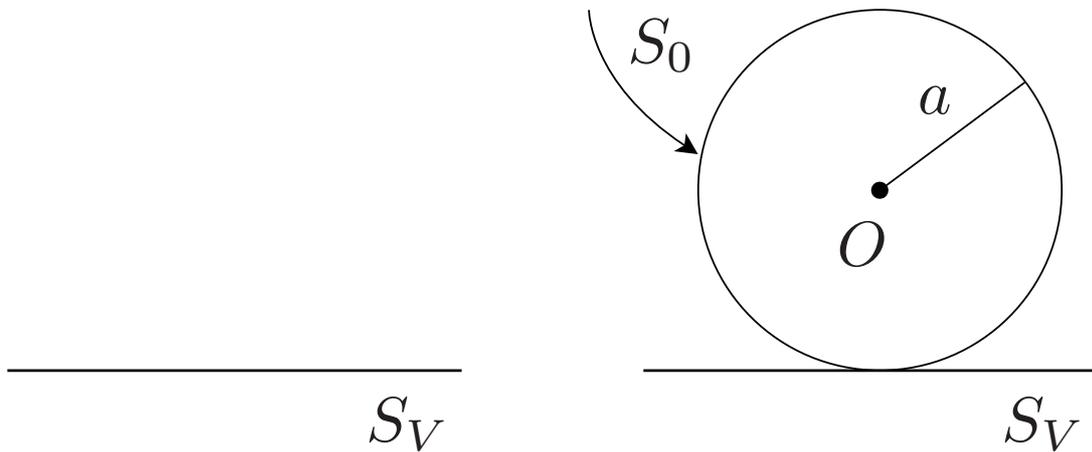


FIG. 1: Illustration of emerging of closed null spacetime system S_0 from vacuum S_V (the idea of picture comes from Fomin's papers [4, 5]).

where $\kappa = 8\pi G/c^4$ and k is a dimensionless parameter. In Fomin's paper λ was assumed to be positive (however we will not restrict it in the further sections). There are some physical reasons for such a choice of the parameterization by considering a heuristic argument based on the conjecture about quantum gravity. Let us suppose that ρ_λ raises due to a quantum gravity fluctuation. Then the action $S_g \propto \int d^4x \sqrt{-g}R$ might be nonzero due to quantum $\Delta V \Delta t$. So $\Delta Dsimeq 1$ [12]. Therefore $\Delta S \propto \frac{\Delta E}{\Delta V} \propto R$ due to energy-time uncertainty relation $\Delta E \Delta t \sim 1$.

III. FOMIN'S MODEL REEXAMINED

Let us assume that a quantum process proposed by Fomin can be modelled, in a phenomenological way, by the energy momentum tensor in the form (2) and when the source of gravity is a perfect fluid with energy density ρ and pressure p . Therefore the Einstein equation assumes the form

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa \bar{T}_{\mu\nu} \quad (4)$$

where $\mu\nu = 0, 1, 2, 3$ and energy momentum tensor is

$$\bar{T}_{\mu\nu} = T_{\mu\nu} - \lambda g_{\mu\nu} = (\rho + p)u_\mu u_\nu + (p - \lambda(x))g_{\mu\nu} \quad (5)$$

$$g = \text{diag}(-1, 1, 1, 1), u_\mu u^\mu = -1, u^\mu = (1, 0, 0, 0), u_\mu = (-1, 0, 0, 0)$$

where $u = (u_0, u_i)$ is a four-vector of velocity.

The form of the Einstein equation (4) is more suitable for our aims because the contribution from the energy density of the vacuum was shifted to the energy momentum tensor. It is clear from divergence of the field equations

$$\bar{T}^{\mu\nu}{}_{;\nu} = \left[R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} \right]_{;\nu} = 0. \quad (6)$$

Hence the energy-momentum tensor is given in the diagonal form

$$\bar{T}_\nu^\mu = (\rho_{\text{eff}} + p_{\text{eff}})u_\mu u_\nu + p_{\text{eff}}g_{\mu\nu} \quad (7)$$

where $p_{\text{eff}} = p - \lambda$, $\rho_{\text{eff}} = \rho + \lambda$ and $\lambda(t) = k\kappa^{-1}R$, $R = \frac{1}{4k-1}\kappa T$, $T = T^\mu{}_\mu = 3p - \rho$ which is obtained following Fomin's assumption from the basic equation

$$R_{\mu\nu} - \left(\frac{1}{2} - k \right) Rg_{\mu\nu} = T_{\mu\nu} \quad (8)$$

with the natural system of units $8\pi G = c = 1$.

Since we are going to construct the cosmological model, as the first approximation, we consider the model, which is spatially homogeneous and isotropic, therefore its spacetime metric is given in the form

$$ds^2 = dt^2 - a^2(t)[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)] \quad (9)$$

where t is the cosmological time; $a(t)$ is a scale factor; r, θ, ϕ are standard spherical coordinates; $\chi: \chi = \arcsin r$, $d\chi = \frac{dr}{\sqrt{1-r^2}}$, because it is assumed a positive value of curvature spatial slices of constant time of hypersurfaces.

In Fomin's original paper, the evolution of the spacetime is governed by a modified Einstein equation rather than the modified energy-momentum tensor (7). In our extension of Fomin's model we shift the contribution coming vacuum instability to the energy-momentum tensor. While both formulations are formally equivalent (gives the same formula $H(z)$), the presented approach offers the possibility of adding the conservation condition to the basic equation, which makes the model in a closed form. Due to homogeneity and isotropy we have $\lambda(x) = \lambda(t)$ and dynamics is equivalent to the dynamics of FRW cosmological models with some effective energy density ρ_{eff} and pressure p_{eff}

$$\rho_{\text{eff}} = \rho + \lambda(x) = \rho + \frac{k}{4k-1}(3p - \rho) \quad (10)$$

$$p_{\text{eff}} = p - \lambda(x) = p - \frac{k}{4k-1}(3p - \rho) \quad (11)$$

$$\lambda(x) = \frac{k}{4k-1}(3p - \rho). \quad (12)$$

Note that if we treat $\rho_\lambda = \lambda(x)$ as an energy density of dark energy then

$$\frac{\rho_\lambda}{\rho} = \frac{\Omega_\lambda}{\Omega_m} = \frac{k}{1-4k} = \text{const.}$$

This means that so-called coincidence problem (i.e. why dark matter and dark energy are of the same order today) is solved naturally in the framework of the Fomin model. If we require

$$\frac{\Omega_\lambda}{\Omega_m} \simeq \frac{0.73}{0.28} = \frac{k}{1-4k}$$

then it is satisfied for $k = 0.22$.

Therefore if we choose the form of the equation of state for matter $p = p(\rho)$ then one can obtain effective values of pressure and energy density. As a result we obtain that the Fomin model belongs to a category of kinematical model with a dynamic value of cosmological constant. Obviously, a solution with dynamic cosmological constant is possible only if $T^{\mu\nu} \neq 0$ (and $T^{\mu\nu}{}_{;\nu} \neq 0$). In the absence of matter ($\rho = p = 0$), λ has got to remain a constant. [13] denoted that this category models which are invoked to solve a cosmological constant problem are in fact consistent with Mach's ideas because an empty spacetime cannot be a solution of general relativity with dynamic cosmological constant ($\lambda(t)$ in our case). It is a consequence of the fact that a conserved quantity is of usual matter and vacuum (and not these two separately) following $\bar{T}^{\mu\nu}{}_{;\nu} = 0$ which reduces to the following equation

$$\frac{d}{dt}(\rho_{\text{eff}}a^3) + p_{\text{eff}}\frac{d}{dt}a^3 = 0 \quad (13)$$

or

$$\left[p\frac{d}{dt}a^3 + \frac{d}{dt}(\rho a^3) \right] + \left[\frac{d}{dt}(\lambda a^3) - \lambda\frac{d}{dt}a^3 \right] = 0. \quad (14)$$

Among the dynamical models of the cosmological constant (for review see [14–17]) there are different choices of cosmological constant parameterization. In the Fomin model the dependence of $\lambda(a(t))$ is determined by the form of the equation of state $p = p(\rho)$ through equation (14). Some different constraints on a decaying cosmological term from the astronomical observation have been found, e.g. [18]. As a consequence the Fomin model with the decaying term $\lambda(a)$ can be tested through the different astronomical observations like distant type Ia supernovae data and Wilkinson Microwave Anisotropy Probe (WMAP) data as well as compared with the concordance Λ CDM model. One can say that the Fomin model is a prototype of the model that incorporates a cosmic time variation of the cosmological term which plays a crucial role in the mechanism of creation of the universe from the vacuum. If $\lambda(a)$ is positive then $k \in (0, 1/4)$.

The evolution of the universe is governed by two basic equations which constitute the closed system of basic equations for the FRW cosmology (with incorporated Fomin's mechanism of gravitational instability of vacuum) when we postulate dependence of energy density and pressure on the scalar factor ($\rho = \rho(a(t))$, $p = p(a(t))$, $\lambda = \lambda(a(t))$)

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho_{\text{eff}} + 3p_{\text{eff}}) = -\frac{1}{6}(\rho + 3p) + \frac{\lambda}{3} = -\frac{1}{6}(\rho + 3p) + \frac{k}{3(4k-1)}(3p - \rho). \quad (15)$$

and the conservation condition (14). To integrate (14) and (15) the form of $p = p(\rho)$ should be postulated. Hereafter for simplicity of presentation we assume that

$$p = \gamma\rho \quad (16)$$

where $\gamma = \text{const}$.

Of course equation (15) possesses the first integral (called the Friedmann first integral)

$$\rho_{\text{eff}} - \frac{3K}{a^2} = 3\frac{\dot{a}^2}{a^2} \quad (17)$$

where $K \in \{0, \pm 1\}$ is a curvature constant. One can simply check this by differentiation of both sides of equation (17) with respect of cosmological time and substitution (17).

The model under consideration contains a new term in comparison with the standard FRW perfect fluid cosmology which is related to the existence of the parameter k not equal zero. This parameter measures the effectiveness of the process of creation of the Universe from the null system. Note that in the radiation epoch, when $p = \rho/3$, the effects of gravitational instability of vacuum vanish. We assume following Fomin that quantum effects of creation of the Universe are manifested at the epoch in which $\rho - 3p$ is positive (this condition is distinguished from Fomin's condition of positiveness of $\lambda(a)$). Therefore, if $0 \leq k \leq \frac{1}{4}$, this guarantees the positiveness of $\lambda(a(t))$. Fomin pointed out that effect of dissociation of vacuum originates in the nonhomogeneous region where $\rho - 3p$. During evolution of the Universe, the effects of vacuum instability stay small because of local expansion of the space.

With the assumption (16) the solution of equation (15) takes the form

$$\rho_{\text{eff}} = \frac{3k(\gamma + 1) - 1}{4k - 1}\rho \quad (18)$$

$$p_{\text{eff}} = \frac{k(\gamma + 1) - \gamma}{4k - 1}\rho \quad (19)$$

From (14) we obtain as a solution

$$\rho_{\text{eff}} = \rho_0 a^{-3(1+w_{\text{eff}})} = \rho_0 a^{-3\frac{4k(\gamma+1)+(k-1)\gamma-1}{3k(\gamma+1)-1}}. \quad (20)$$

(In the special case of dust matter we obtain $\rho_{\text{eff}} = \rho_0 a^{-3\frac{4k-1}{3k-1}}$.)

Hence we determine the equation of state coefficient

$$w_{\text{eff}} \equiv \frac{p_{\text{eff}}}{\rho_{\text{eff}}} = \frac{k(\gamma + 1) - \gamma}{3k(\gamma + 1) - 1}. \quad (21)$$

We obtain from formula (21) that the Universe is accelerating for dust filled matter if the parameter k belongs to the interval $k \in (\frac{1}{6}, \frac{1}{3})$; $w_{\text{eff}} < -\frac{1}{3}$.

For our further analysis of constraints from SNIa observational data it is useful to represent an evolutionary scenario of models in terms of the Hubble function and dimensionless density parameters. For the two distinguished cases we obtain

$$H^2 = \frac{\rho_{\text{eff}}}{3} - \frac{1}{a^2} = \frac{\rho_{\text{eff}}(z)}{3} - \frac{1}{(1+z)^2} \quad (22)$$

where $1+z = \frac{a_0}{a}$; $a_0 = 1$ is the present value of the scale factor and

$$\frac{H^2}{H_0^2} = \Omega_{K,0}(1+z)^2 + \Omega_{m,0}(1+z)^3\frac{4k(\gamma+1)-\gamma-1}{3k(\gamma+1)-1}$$

where $\Omega_{i,0} = \frac{\rho_{i,0}}{3H_0^2}$ are the density parameters for the i -th component of the fluid. Of course $\sum_i \Omega_{i,0} = 1$ from condition $H(z=0) = H_0$. In the special case of dust matter we obtain the useful relation in the context of constraining the model parameters against the astronomical data

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{K,0}(1+z)^2 + \Omega_{m,0}(1+z)^{3\frac{4k-1}{3k-1}} \quad (23)$$

where $\Omega_{K,0} + \Omega_{m,0} = 1$.

We obtain that universe is still accelerating if $w_{\text{eff}} < -1/3$ and $\rho_{\text{eff}} > 0$, i.e.

$$\frac{6k(\gamma+1) - 3\gamma - 1}{3k(\gamma+1) - 1} < 0 \quad \text{and} \quad \frac{3k(\gamma+1) - 1}{4k - 1} > 0.$$

For dust it means that that

$$\frac{6k - 1}{3k - 1} < 0 \quad \text{and} \quad \frac{3k - 1}{4k - 1} > 0$$

i.e. that $k \in (\frac{1}{6}, \frac{1}{4})$. Therefore the Fomin model predicts acceleration of the universe driven by the mechanism of origin of the universe from gravitational instability of vacuum.

In the next section we concentrate on the constraints from recent SNIa measurements of the effects arising from possible Fomin's quantum creation mechanism which can be phenomenologically manifested by dark energy through a modified Friedmann equation.

IV. DISTANT TYPE IA SUPERNOVAE AS COSMOLOGICAL PROBES OF DARK ENERGY

Today cosmology appears to be one of the fastest growing parts of physics due to new experiments from measurements of the cosmic microwave background radiation to measurements of the apparent magnitudes of several high redshift supernovae of type Ia, published recently by Riess et al. [19]. For distant supernovae, one can directly observe the apparent magnitude (i.e., log of flux F) and its redshift. As the absolute magnitude M of any supernovae is related to the present luminosity L , then relation $F = \frac{L}{4\pi d_L^2}$ can be written as

$$m - M = 5 \log_{10} \left(\frac{d_L}{\text{Mpc}} \right) + 25. \quad (24)$$

Usually instead of d_L dimensionless quantity $\frac{H_0 d_L(z)}{c}$ is used, and then equation (23) changes to

$$m(z) = \bar{M} + 5 \log_{10} \left(\frac{H_0 d_L(z)}{c} \right) \quad (25)$$

where the parameter \bar{M} is related to M by the relation

$$\bar{M} = M + 25 + 5 \log_{10} \left(\frac{c H_0^{-1}}{\text{Mpc}} \right). \quad (26)$$

We know the absolute magnitude of SNIa from its light curve. Therefore we can obtain d_L for these supernovae as a function of redshift because the apparent magnitude M can be determined from low z apparent magnitude. Finally, it is possible to probe dark energy which constitutes main contribution to the matter content from $d_L(z)$. In our further analysis of SNIa data, we estimate models with value of $M \cong 15.955$ determined from low redshift relation ($d_L(z)$ is then linear) without any prior assumption on H_0 .

We use the standard statistical approach to obtain the best fitting model minimizing the χ^2 function. This analysis is supplemented by the maximum likelihood method to find confidence ranges for the estimated model parameters.

It is assumed that supernovae measurements came with uncorrelated Gaussian errors and the likelihood function \mathcal{L} could be determined from the χ^2 statistic $\mathcal{L} \propto \exp\left(-\frac{\chi^2}{2}\right)$. In our analysis the latest compilation of SNIa prepared by Riess et al. is used (the Gold sample) [19].

Recent measurements of type Ia supernovae as well as other WMAP and extragalactic observations, suggest that the expansion of the Universe is an accelerating expansion phase. There are different attempts to explain the present

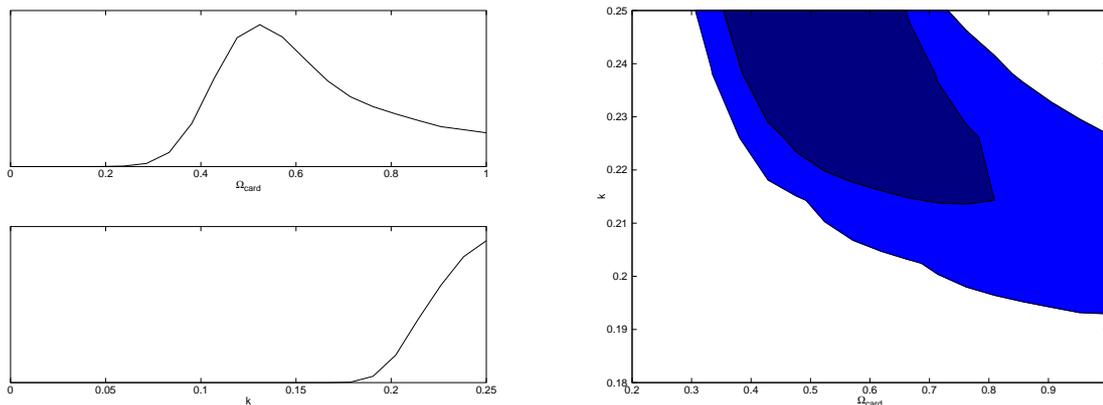


FIG. 2: The estimation of parameters $\Omega_{\text{Card},0}$ and k ; left panel – posteriori probability distributions; right panel – posteriori probability confidence levels 68% and 95%.

acceleration of the Universe. Dark energy of unknown form has usually been invoked as the most feasible mechanism. Also effects arising from exotic physics, like extra dimensions, some modification of the FRW equation can also mimic dark energy through a modified Friedmann equation.

It would be interesting to check the consistency of Fomin's model with SNIa observations. We begin by evaluating the luminosity distance as a function of redshift z as well as the parameters of the model. We define the redshift dependence of H as $H(z) = H_0 E(z)$. For Cardassian stylization of Fomin's model of the closed universe with matter (baryonic and cold matter) we get that the luminosity distance is given by

$$d_L(z, \Omega_{\text{m},0}, H_0) = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{E(z')}. \quad (27)$$

The standard measure of acceleration in cosmology is the dimensionless deceleration parameter q defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \quad (28)$$

We can calculate the value of this parameter for both classes of models under consideration. Then we obtain

$$q_0 = \frac{1}{2}(1 + 3w_{\text{eff}}) = \frac{4k(\gamma + 1) - \gamma - 1}{2[3k(\gamma + 1) - 1]}. \quad (29)$$

We estimate the two-parameter model in the following form

$$\left(\frac{H}{H_0}\right)^2 = (1 - \Omega_{\text{card},0})(1+z)^2 + \Omega_{\text{Card},0}(1+z)^{3n} \quad (30)$$

where $\Omega_{\text{card},0}$ is positive and less than one, and the parameter n is positive. The latter parameter is related to the searched parameter k

$$n = \frac{4k - 1}{3k - 1} \quad (31)$$

where we assume k to be $(0, 1/4)$.

Using the SNIa Union sample we estimate both parameters $\Omega_{\text{Card},0}$ and k to find

$$\Omega_{\text{Card},0} = 0.62_{-0.15}^{+0.22} \quad k = 0.23_{-0.01}^{+0.01}$$

where the 68% confidence levels are in brackets. These posteriori probability distributions are shown in Fig. 2.

V. DYNAMICS OF THE COSMOLOGICAL MODELS WITH FOMIN'S MECHANISM CREATION OF THE UNIVERSE

The dynamics of the model can be represented with the help of a particle-like description, namely it can be reduced to the form of a particle of unit mass moving in a potential well. The forms of potential function for dust matter and matter with $p = \gamma\rho$ models respectively are

$$V(a) = -\frac{1}{2} \left(\Omega_{K,0} + \Omega_{m,0} a^{-3\frac{4k-1}{3k-1}+2} \right) \quad (32)$$

$$V(a) = -\frac{1}{2} \left(\Omega_{K,0} + \Omega_{m,0} a^{-3\frac{4k(\gamma+1)-\gamma-1}{3k(\gamma+1)-1}+2} \right). \quad (33)$$

There are two different scenarios of initial state of the Universe namely a singularity or a bounce. It depends on the exponent in the potential form (32). Note that if $-\frac{3(4k-1)}{3k-1} + 2$ is negative this term will dominate the curvature term and we have a standard singularity (V goes to minus infinity). This happens when

$$\frac{-6k(\gamma+1) + 3\gamma + 1}{3k(\gamma+1) - 1} < 0.$$

For dust matter it is satisfied if $k \in (-\infty, 1/7) \cup (1/4, \infty)$.

In the opposite case if $k \in (0, 1/7)$ we obtain a bounce as a scenario of an initial state. This scenario is the generic feature of cosmology inspired by quantum gravity effects [20].

Note that for $\gamma = 1/3$ effects of decaying λ vanish and the model evolution follows the radiation dominating universe scenario.

The motion of the system is defined on the zero energy level $H = \frac{\dot{a}^2}{2} + V(a) \equiv 0$. In the bouncing scenario the standard evolution with a singularity at $a = 0$ is replaced by a bounce. Note that negative contribution to $H^2(a)$ formula cannot dominate the material term and the Universe starts from some $a = a_{\min} > 0$ — a finite value of the scale factor. On the other hand, evolution can be prolonged into the domain $t < 0$ (a pre-bounce region) because of the reflectional symmetry $H \rightarrow -H$. If $\Omega_{K,0} < 0$, i.e. the Universe is closed, then we obtain in the configuration space returning points. As a consequence we obtain characteristic type of evolution without initial singularity. Let us illustrate this situation. For the model with dust we have

$$V(a) = -\frac{1}{2} \left[\Omega_{K,0} + (1 - \Omega_{K,0}) a^{\frac{-6k+1}{3k-1}} \right] \leq 0.$$

This function has always zero: $a = a_0 = \left(-\frac{\Omega_{K,0}}{1 - \Omega_{K,0}} \right)^{\frac{3k-1}{-6k+1}}$ and $a \geq a_0$ for $k \in (1/6, 1/3)$ and $a < a_0$ otherwise. The derivative of $V(a)$ determines a domain of acceleration

$$\ddot{a} = -V'(a) = -\frac{1-6k+1}{2} \frac{1-6k+1}{3k-1} (1 - \Omega_{K,0}) a^{\frac{2-9k}{3k-1}}.$$

Therefore for any t the scale factor acceleration $\ddot{a} > 0$ takes place if $V'(a) < 0$ i.e., $k \in (1/6, 1/3)$. In the opposite case $V(a)$ is a growing function of a ($V'(a) > 0$) and $k \in (0, 1/6) \cup (1/3, \infty)$. At $a = a_0$ $H = 0$ because $\dot{a}^2 = -2V$. If $(-6k+1)/(3k-1) > 0$ then $V(a_0) = 0$ and it is a bouncing point at a_0 . In the opposite case $V(a=0) = -\infty$, i.e. there is a singularity at a_0 . Summing up, in the acceleration regime V is a decreasing function of a , $k \in (1/6, 1/3)$, and there is a bouncing point at a_0 . In the opposite, deceleration regime V is an increasing function of a , $k \in (0, 1/6) \cup (1/3, \infty)$ and there is an initial singularity and a maximum size of the universe a_0 .

The phase portrait of model for two cases is in Fig. 3.

In the literature there are well known propositions of explanation of acceleration of the Universe using cosmological models with modified Friedmann equations. Freese and Lewis [21] showed that the Cardassian modification of Friedmann equation (which contains a term of type $\Omega_{\text{Card},0}(1+z)^{3n}$, $n = \frac{4k-1}{3k-1}$) may give rise to acceleration without any dark energy of unknown origin contribution. In Fomin's model the age of the bouncing universe ($k \in (1/6, 1/3)$) in the approximation of small $\Omega_{K,0}$ is given by formula

$$T = \frac{2}{3} H_0^{-1} \frac{3k+1}{1-4k}.$$

Therefore $k \simeq 1/6$ is required to explain the problem of age of the Universe.

The main goal of this section is to set observational constraints on this additional parameter k and then check whether it is consistent with current SNIa cosmological data. We also perform analogous analysis for the Cardassian model [22].

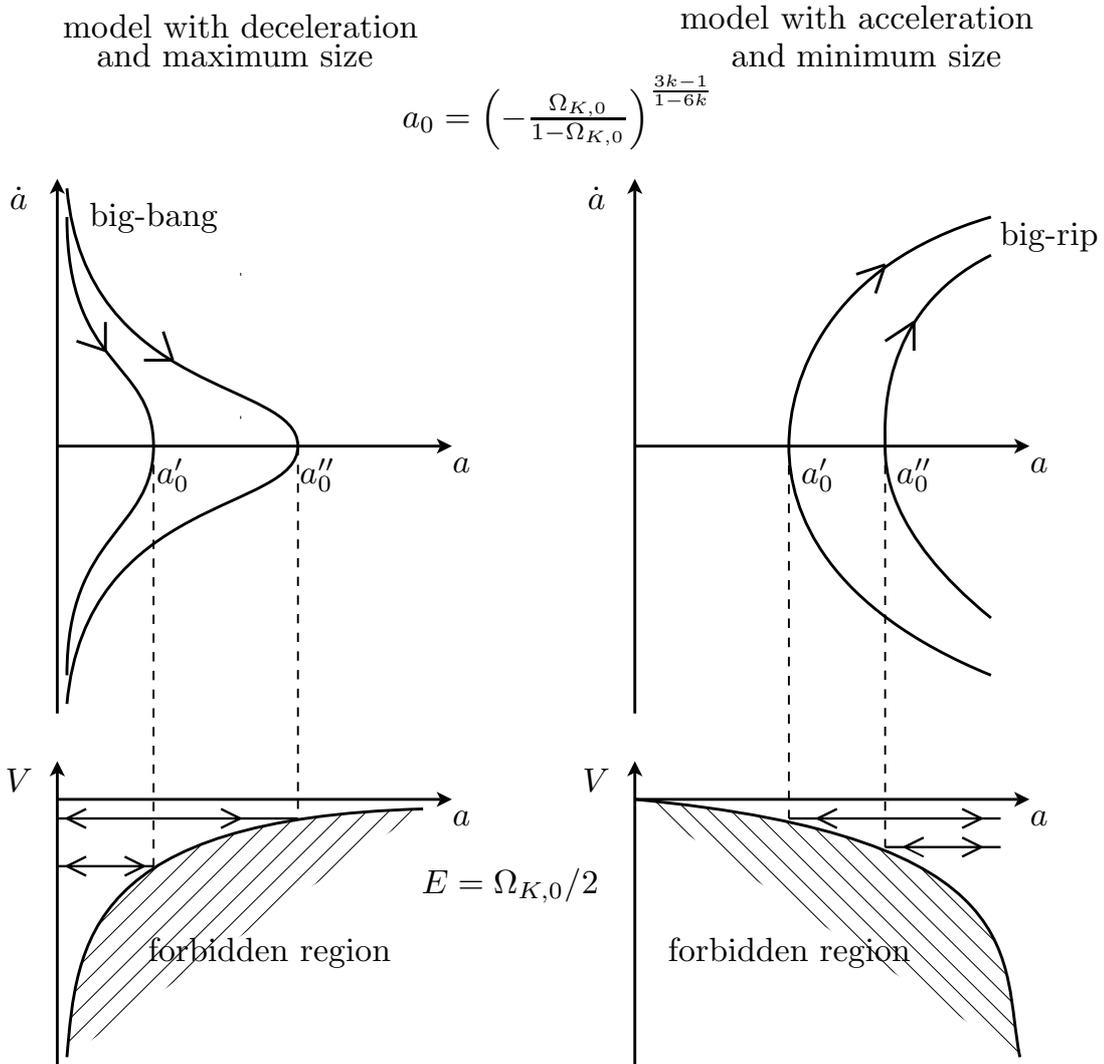


FIG. 3: The phase portrait and shape of diagram of potential function for Fomin model.

VI. CONCLUSIONS

We reviewed, clarified, and critically analyzed early approaches to Tryon's and Fomin's description of the origin of the Universe. We showed that the idea of the Universe's creation as a vacuum fluctuation event was discovered by them independently at around the same time. While Tryon's idea seems to be interesting only from the historical point of view, Fomin's approach is still heuristically fertile in description of latest issues in cosmology. Fomin's model brings the interesting explanation of the accelerating expansion of the Universe in terms of the Ricci scalar dark energy $\rho_X \propto R$ [23–26]. This idea is important in the context of holographic principle of quantum gravity theory which regards black holes as the maximally entropic objects. For a given region of size L effective field theory with UV cut-off Λ gives the Bekenstein entropy bound $(L\Lambda)^3 \leq S_{\text{BH}}$ – entropy of a black hole.

In this paper we study Fomin's idea of quantum cosmogenesis through the dynamical analysis. It is possible, in contrast to Tryon, because Fomin's conception is embedded in the environment of general relativity rather than in the Newtonian theory. We demonstrated that it is possible to construct the dynamical cosmology with incorporation of Fomin's mechanism of gravitational decaying of vacuum. This cosmology can be treated as a cosmology with varying cosmological constant. In Fomin's model appears the free parameter k . There are some interval of this parameter for which the model accelerates and possesses a bounce instead of an initial singularity. It is necessary then to estimate it from the astronomical observations.

As recent supernovae of type Ia measurements indicate the expansion of our Universe is presently accelerating. While the cosmological constant offers the possibility of reconstruction of the effective theory of acceleration, the presence of

fine-tuning difficulties motivates theorists to investigate alternative forms of dark energy. All these propositions can be divided into two groups. In the first group, unusual properties of dark matter with negative pressure violating the strong energy condition are postulated. In the other one, the modification of the Friedmann equation is postulated. In this approach, instead of a new hypothetical energy component of unknown form, some modifications of the FRW equations are proposed a priori (unfortunately without any fundamental justification).

The status of Fomin's cosmology seems to be similar to the status of the brane cosmological models where our observable Universe is a surface or a brane embedded in a higher dimensional bulk spacetime in which gravity could spread. Note, that this model with radiation can be recovered if parameter k is $1/5$. We point out that Fomin's idea can be tested (in contrast to Tryon's idea) by using type Ia supernovae data.

It is also important that Fomin's approach is free from the assumption of zero energy which is crucial for Tryon. Recently many authors concluded that energy-momentum tensor of the FRW universes are equal to zero locally and globally (flat universe) or just globally (closed universe). However, such a conclusion originated from coordinate-dependent calculations performed in the special comoving coordinates (Cartesian coordinates).

It is interesting that predictions of this theory can be also tested by both Wilkinson Microwave Anisotropy Probe (WMAP) observations as well as by observations of distant supernovae type Ia (SNIa). We confront the Fomin FRW model with SNIa observation and estimate all parameters of models. Our analysis showed that Fomin's idea plays not only a historical role but is related to the modern cosmological models appeared in the context of dark energy.

The idea that sum of energies of all particles in the Universe is the same order of magnitude like gravitational energy, and consequently the total energy vanishes is very old. Overduin and Fahr [27] argued that Haas [28] and Jordan [29, p.16] introduced this idea in 1936 and 1947, respectively, and called it the Haas-Jordan idea. The main difficulty in extending it to relativistic cosmology is caused by finding the correct definition of the energy in the FRW universe. The problem was not solve so far. On the other hand it was proposed to use the Newtonian framework to show this idea at work. But this approach basing on the concept of infinite R^3 space as a model of real space is inconsistent with homogeneous and isotropic static distribution of matter.

The FRW general relativistic cosmological models can be represented as a motion of a fictitious particle moving in the potential well [11]. The shape of the potential function is determined by both matter content and curvature. In the simplest case the potential function is a function of the scale factor only. The balance energy relation $T + V = 0$ corresponds to the Hamiltonian energy constraint $H = 0$. Note that division on kinetic and potential parts of energy has purely conventional character. The system under consideration has a natural Lagrangian. From the potential function the curvature term can be extracted in such a way that motion of the system is restricted to the energy level $H = E = -\frac{K}{2}$. In the Newtonian cosmology the kinetic part is zero and the solution is admissible only for a closed universe because the gravity is attracting. Hence the problem of extension of the Haas-Jordan idea for the general relativistic case seems to be crucial for deeper understanding of the cosmogenesis mechanism.

The weakness of the Fomin model is that it does not specify what kinds of matter are created. If you have several components, you get only the overall balance equation, but not how the created matter is divided between the components. However the modern cosmological context of Ricci scalar dark energy suggests that created energy may corresponds to dark energy component.

From the philosophical point of view it means that there is some class of models of quantum cosmogenesis which are susceptible to falsify. We obtain from observation that value of the basic parameter k is 0.23 and $\Omega_{\text{Card},0} = 0.62$ as the best fit. We also calculated the PDFs and confidence level of two model parameters Our analysis showed that in contrast to the Tryan model the Fomin model can be statistically estimated and in the light of SNIa data can explain accelerating phase of expansion of the current Universe.

Acknowledgments

The authors are very grateful to prof. M. Heller and dr A. Krawiec for remarks and comments. Marek Szydlowski is especially very grateful to prof. Y. Shtanov for information on a detailed publication history of Fomin's paper and the fruitful discussion on quantum cosmogenesis. The authors also thank prof. J. Overduin and prof. A. Vilenkin for comments and historical remarks. This work was supported in part by the Marie Curie Actions Transfer of Knowledge project COCOS (contract MTKD-CT-2004-517186) and Polish State Committee for Scientific Research (KBN) grant No. 1 H01A 019 30.

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