Effects of delay in knowledge development

Marek Szydłowski [1,3] and Adam Krawiec [2,3]

[1] Institute of Philosophy of Nature and Natural Sciences, John Paul II Catholic University in Lublin
[2] Institute of Economics and Management, Jagiellonian University
[3] Mark Kac Complex Systems Research Centre, Jagiellonian University
uoszydlo@cyf-kr.edu.pl, uukrawie@cyf-kr.edu.pl

Abstract

We generalize the dynamical models of accumulation scientific results by introducing time delay to the standard de Sola Price model. The delay is interpreted as the time required for learning and understanding a new theory [5, 6]. As a result we obtain cyclic behaviour, which we arise in differential equation with time delay, due to the Hopf bifurcation. We estimate the period of cycles. We also show the existence of growth cycle in knowledge development on the example of symbolic logic.

1. Introduction

Discussing the science development de Solla Price introduced the hypothesis of an exponential growth of scientific results and verified it on a large historical data set [3]. His research opened discussions in the scientometrics community and as a consequence the quantitative investigation of science became popular [4].

We propose a model of growth of a given domain of science where the growth is measured by the number of important publications appearing at a given time. When testing the model the importance of a scientific publication should be determined in an operational sense, for instance, on the basis of the Citation Index or by the decree of a scientific authority, as it was the case when Alonso Church composed a list of publications in symbolic logic for the period 1666-1934.

We assume that the number of important publications determines the state of science at a given time. We use the term "state of science", but we mean the state of a given branch of science, or a research program, or a given theory, depending on the modelled domain. The growth of science has a continuous character but there exists a finite time period T needed to build up a result of the fundamental character. Further we assume that T is constant.

2. The model with a delay time and constant rate growth of number of scientists

As is well known, various economic and social processes can successfully be modelled in terms of systems of differential equations called dynamical systems. Since the process of growing science is similar to other economic and social processes (it is, in fact, itself a social process) as far as the degree of its complexity is concerned, treating it as a dynamical system seems naturally.

Based on the assumption that new scientific results stand on the existing knowledge and require time to be discovered, analysed, tested, and written down the simple dynamical system in a linear form is postulated

$$\dot{x}(t) = \alpha x(t-T) \tag{1}$$

where $x(t)$ is the number of papers in t and T is a constant time delay required for new result to appear. The de Solla Price model is obtained when the delay is zero. Let us introduce the change of population of scientists as an important factor in the growth of scientific field. We assume that the population grows at the constant rate n ($\dot{L} = nL$). Let us introduce the variable $z = x/L$ then the model has the form

$$\frac{dz}{dt} = \alpha z(t-T) - nz(t). \tag{2}$$

This equation describes the evolution of the number of papers written by a scientist. The above equation is still linear but now the cyclic behaviour may appear because of the feedback between the delayed and non-delayed terms.

To show the existence of the cyclic behaviour in this model we use methods of the bifurcation theory. The role of a control parameter in the model is played by the delay parameter T. First, we consider the characteristic equation

$$\lambda = \alpha e^{-\lambda T} - n \tag{3}$$

Next we assume that the eigenvalue is complex $\lambda = \sigma + i\omega$. By inserting $\lambda = \sigma + i\omega$ we obtain

$$\sigma = \alpha e^{-\sigma T}\cos\omega T - n \tag{4a}$$
$$-\omega = \alpha e^{-\sigma T}\sin\omega T. \tag{4b}$$

The bifurcation value $T = T_{\mathrm{bi}}$ leading to periodic orbit is obtained by substituting $\sigma = 0$. Then, system (4) can be solved in an exact form

$$\omega_{\mathrm{bi}} T_{\mathrm{bi}} = -\arctan\left(\frac{\omega_{\mathrm{bi}}}{n}\right) + j\pi \tag{5a}$$
$$\omega_{\mathrm{bi}} = \pm\sqrt{\alpha^2 - n^2}. \tag{5b}$$

Let us notice that these equations are symmetric with respect to the reflection $\omega = -\omega$ which means that if λ is a solution then $\bar{\lambda}$ is a solution as well. Therefore, we may assume $\omega > 0$. From these equations we can find $\omega(T_{\mathrm{bi}})$, and then the period $P = \frac{2\pi}{|\omega(T_{\mathrm{bi}})|}$. The period of the principal cycle ($j = 0$) is given by

$$P \simeq \frac{2\pi}{\sqrt{\alpha^2 - \beta^2}}. \tag{6}$$

We can also show that the period of the principal cycle is distinguished, because it is the only period longer than the bifurcation parameter value. The interpretation of minor cycles with periods shorter then the bifurcation parameter value is unattainable.

We can find the dependence between the bifurcation time T_{bi} and the period of the principal cycle P. If $\alpha \gg n$ then

$$P = 4T_{\mathrm{bi}}.$$

From eq. (5) we find that the bifurcation time T_{bi} (if $n = 0$) is given by

$$T_{\mathrm{bi}} = \frac{\pi}{2\alpha}.$$

In general

$$T_{\mathrm{bi}} = \frac{1}{\alpha\sqrt{1-\left(\frac{n}{\alpha}\right)^2}}\arctan\sqrt{\left(\frac{\alpha}{n}\right)^2 - 1}. \tag{7}$$

The graph presenting T_{bi} as the function of α, for $n/\alpha = 1/3$ is shown in Fig. 1. As it can be observed, to generate a long-term cycle one must assume relatively large delay time; e.q., for $T_{\mathrm{bi}} = 25$ years (this is approximately the length of the active period in the life of a scientist), one obtains $P = 100$ years. The parameter n diminishes this value.

We can see that, for a value of T from the suitable neighbourhood of T_{bi}, the system has a periodic solution which in the phase space is represented by the isolated periodic orbit of a given period because the system considered is linear.

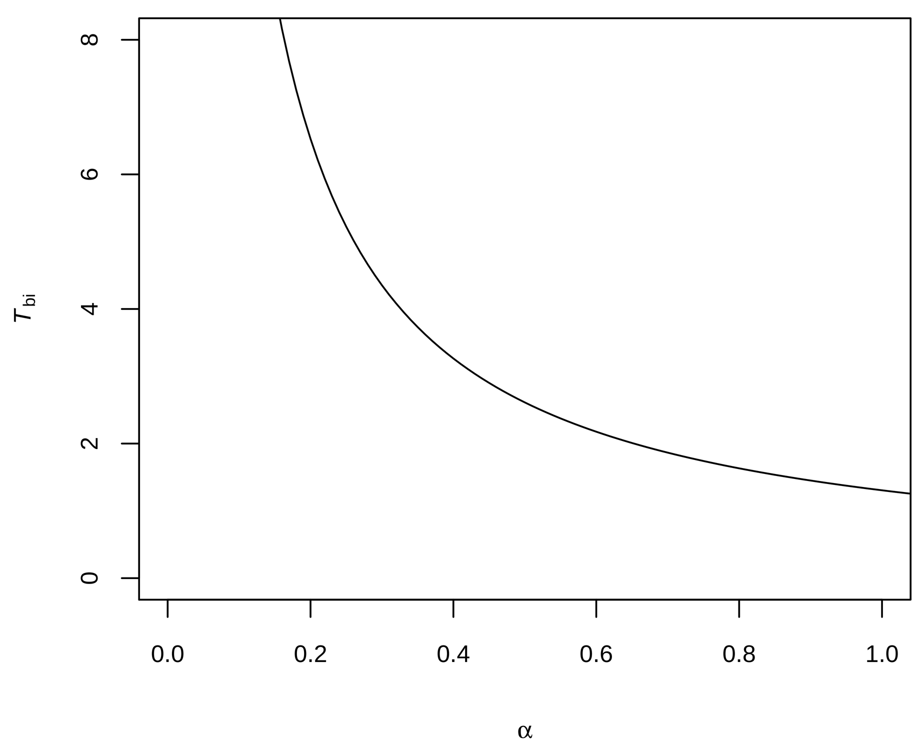

Figure 1: *The graph of T_{bi} with $n/\alpha = 1/3$ as the function of α given by (7).*

3. Analysis of bibliographic data

In the analysis of bibliographic data we consider a linear model with time delay where the cumulative total number of papers x is considered

$$\frac{dx}{dt} = \alpha x(t-T) \tag{8}$$

where α is a proportionality constant. The initial condition $y(t) = \phi(t)$ is defined on the interval $[-T, 0]$. Equation (8) states that the rate of science growth is proportional to the number of publications at the past time $t - T$ where the time period T is required to obtain, work up a new result. It is very well known from the history of science that a certain time must elapse between the first appearance of a given scientific concept or method and its full exploitation in science.

It is interesting that it is possible to obtain the exact solution of (8) with $\alpha > 0$ in terms of eigenvalues of the linearisation matrix. The characteristic equation is of the transcendental form and admits the infinite number of complex solutions

$$x(t) = C_0 e^{rt} + \sum_{k=1}^{\infty} e^{p_k t}(C_{1k}\cos(q_k t) + C_{2k}\sin(q_k t))$$

where $\lambda_k = p_k \pm iq_k$ are complex roots of the characteristic equation $\lambda = \alpha e^{-\lambda T}$; r is a real root of the characteristic equation; and C_0, C_{1k}, C_{2k} are arbitrary constants. What is unexpected in this model is that it admits oscillations while the system is linear. The first term in the general solution represents the exponential part, while the sum describes fluctuations along this trend. We start with an estimation of this base trend.

We use Church's bibliographic list of works in symbolic logic [1, 2]. The cumulative numbers of works in the period 1666–1934 is presented in Fig. 2. We can estimate the exponential curve from this data. The standard analysis of linear regression for $\ln(x) = b + at$ where x is the cumulative number of works in time t, and b, a are constants, yields

$$\ln(x) = -40.4598 + 0.0243450t \tag{9}$$

with $R^2 = 0.938232$. The regression is shown in Fig. 3. We can see that some oscillations exist around on the exponential trend. In the previous section we argued that the time required to build a result on the old ones is the cause of the oscillation around this exponential trend.

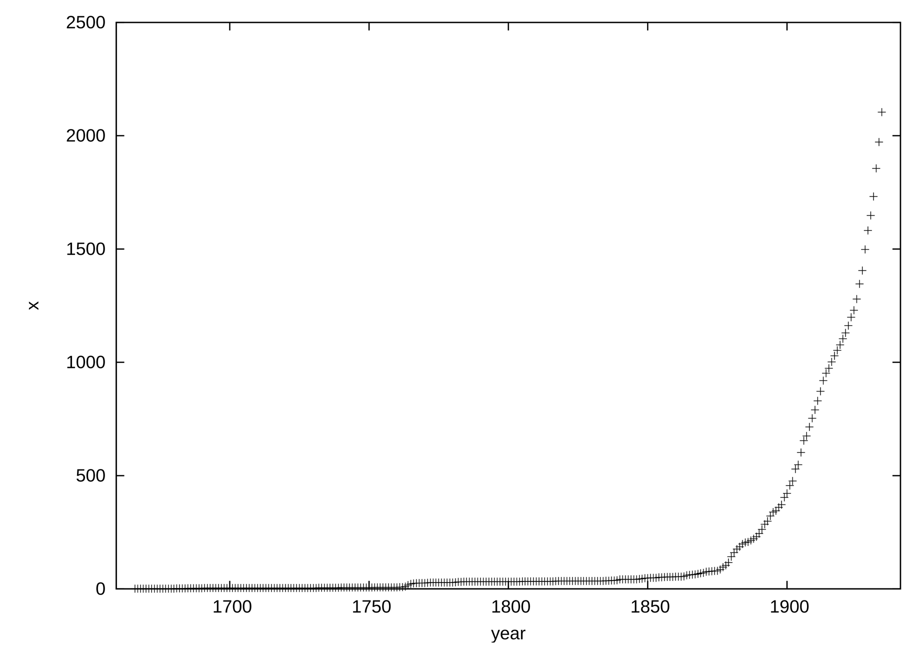

Figure 2: *The cumulative number of symbolic logic works x in period 1666–1934.*

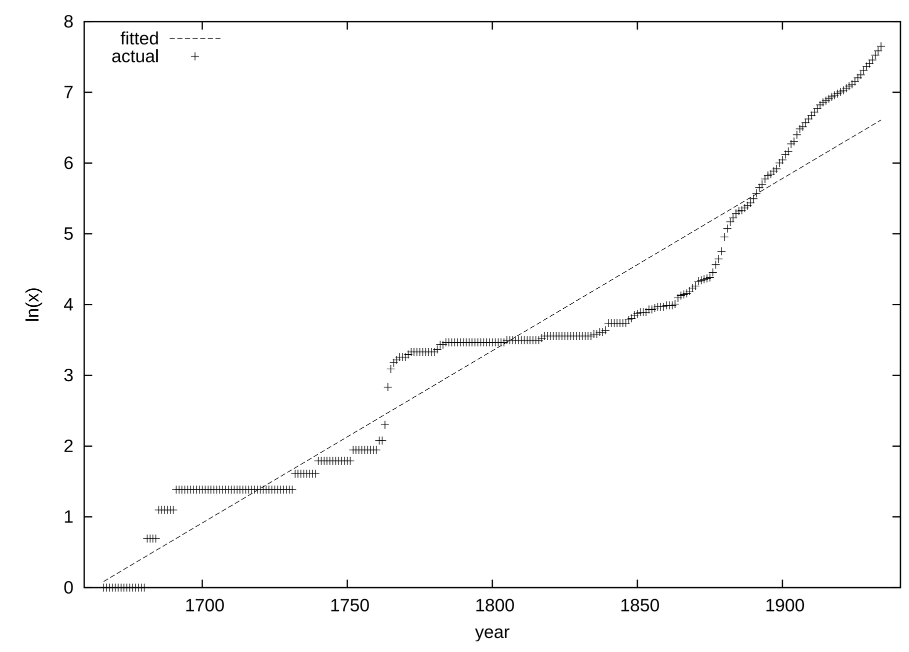

Figure 3: *The logarithm of the cumulative number of symbolic logic works x and the regression line.*

4. Conclusions

We showed that development of knowledge could be seen as an increase of the number of works published in a given field of science. This growth has the exponential character with some irregularities around the trend. As the cause of this fluctuations we considered the time delay interpreted as time passed between the publishing of a fundamental, important paper and papers based on it. We constructed a model of the growth of a field of science in which the growth is measured by the number of important works. We showed that in this model there exists a mechanism of a cyclic behaviour. It is a bifurcation scenario leading to periodic orbits. If the time delay is large enough, then the long term cycles appear with the period equal to $4T_{\mathrm{bi}}$.

We considered only the time delay as the main source of oscillations in accumulation of scientific results. For simplicity we neglected the exogenous factors such as social and political events which should cause cycles in the development of science.

References

[1] A. J. Church. A bibliography of symbolic logic. *Journal of Symbolic Logic*, 1:121–218, 1936

[2] A. J. Church. Additions and corrections to A bibliography of symbolic logic. *Journal of Symbolic Logic*, 3:178–192, 1938

[3] D. J. de Solla Price. *Science since Babylon*. Yale University Press, New Haven 1961.

[4] B. C. Griffith. Derek Price (1922–1983) and the social study of science. Scientometrics 6:5–7, 1984

[5] M. Szydłowski, A. Krawiec. Scientific cycle model with delay. *Scientometrics*, 52:83–95, 2001.

[6] M. Szydłowski, A. Krawiec. Growth cycles of knowledge. *Scientometrics*, 78:99–111, 2009.