

KARTA PRZEDMIOTU**I. Dane podstawowe**

Nazwa przedmiotu	Probability theory
Nazwa przedmiotu w języku angielskim	Probability theory
Kierunek studiów	Mathematics
Poziom studiów (I, II, jednolite magisterskie)	I
Forma studiów (stacjonarne, niestacjonarne)	stationary
Dyscyplina	Mathematics
Język wykładowy	English

Koordinator przedmiotu/osoba odpowiedzialna	dr hab. August Zapała
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Forma zajęć (<i>katalog zamknięty ze słownika</i>)	Liczba godzin	semestr	Punkty ECTS
wykład	60	5	9
konwersatorium			
ćwiczenia	45	5	
laboratorium			
warsztaty			
seminarium			
proseminarium			
lektorat			
praktyki			
zajęcia terenowe			
pracownia dyplomowa			
translatorium			
wizyta studyjna			

Wymagania wstępne	Introduction to mathematics, Introduction to differential and integral calculus, Mathematical analysis I and II (sequences and series of numbers, differential and integral calculus of functions of one variable and many variables) Topology of metric spaces (foundations of set theory, properties of compact metric spaces)
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II. Cele kształcenia dla przedmiotu

Understanding of methods for mathematical description of random phenomena
Learning how to calculate probabilities of random events, determine distributions of random variables and evaluate numerical parameters of probability distributions
Understanding of various modes of convergence of sequences of random variables
Understanding of characteristic functions (Fourier transformation).
Understanding of basic limit theorems of probability theory

III. Efekty uczenia się dla przedmiotu wraz z odniesieniem do efektów kierunkowych

Symbol	Opis efektu przedmiotowego	Odniesienie do efektu kierunkowego
WIEDZA KNOWLEDGE		
W_01	Students familiarize with various definitions of probability, the notions of a random variable, distribution function and probability density	K_W03
W_02	Students get acquainted with the most important discrete and continuous probability distributions	K_W03
W_03	Students get acquainted with characteristic functions (Fourier transformation) and inversion formulas	K_W04
W_04	Students get acquainted with the most important limit theorems of probability theory	K_W04
UMIĘTNOŚCI SKILLS		
U_01	Students are capable to build and analyse models of random phenomena by means of appropriate probability spaces and evaluate probabilities of random events	K_U30
U_02	Students can apply the total probability formula and Bayes' formula	K_U32
U_03	Students are capable to give various examples of discrete and continuous probability distributions and they know practical applications of these distributions	K_U31
U_04	Students can determine parameters of distributions of discrete and continuous random variables, find equations of regression lines, calculate characteristic functions, and apply limit theorems and laws of large numbers to estimate probabilities	K_U33
KOMPETENCJE SPOŁECZNE SOCIAL COMPETENCES		
K_01	Students precisely formulate questions to deepen the understanding of the subject and complement the missing elements of reasoning	K_K02

IV. Opis przedmiotu/ treści programowe

The sample space and random events, fields and σ -fields of events. Classical, geometrical and statistical definitions of probability, examples of applications. Axioms of probability, finitely additive probability and axiom of continuity.

Independence of events, fields and σ -fields of events. Conditional probability, the law of total probability and Bayes' formula. Discrete probability spaces. Distribution functions in 1-dimensional and multidimensional Euclidean space. Construction of probability measures from distribution functions in 1-dimensional and multidimensional Euclidean space.

Random variable, the law and distribution function of the random variable. Discrete and continuous distributions, probability mass function and probability density.

Information concerning the Jordan theorem and the Lebesgue-Radon-Nikodym theorem on decomposition of distribution functions.

Random vectors. Multidimensional discrete and continuous distributions, probability densities in multidimensional spaces. Marginal distributions of discrete and continuous

random vectors. Independent random variables, criteria of independence for discrete and continuous random variables.

Numerical characteristics of random variables. Expectation and its properties. Variance, standard deviation, and their properties. Moments and central moments. Covariance and correlation coefficient, properties of the correlation coefficient. Lines of regression.

Various modes of convergence of random variables (in distribution, in probability, almost sure and in mean). A criterion for almost sure convergence. Markov's and Chebyshev's inequalities and certain their applications. Relationships between various modes of convergence.

Complex random variables, independence and expectations of complex random variables. Some useful inequalities for complex random variables.

Characteristic functions and their properties. Lévy's theorem (the inversion formula for distribution functions). Inversion formulas for discrete distributions. Inversion formulas for probability densities.

Helly's lemmas and the Helly-Bray theorem. The Lévy-Cramér theorem.

The Lindeberg-Feller central limit theorem for triangular arrays of random variables.

The Lindeberg-Feller central limit theorem for a sequence of random variables, Lyapunov's and Lindeberg-Lévy theorems.

Weak law of large numbers – Khintchine's, Chebyshev's and Markov's theorems.

Theorem on almost uniform convergence of a sequence of characteristic functions.

Inequalities for truncated random variables. The notion of median and Levy's symmetrization inequality. Classical criterion for convergence to a constant.

The Borel zero-one law and the Borel-Cantelli lemma. Kolmogorov's inequality, Kolmogorov's criterion and the Kolmogorov strong law of large numbers.

V. Metody realizacji i weryfikacji efektów uczenia się

Symbol efektu	Metody dydaktyczne (lista wyboru)	Metody weryfikacji (lista wyboru)	Sposoby dokumentacji (lista wyboru)
WIEDZA			
W_01	Conventional lecture	Test / Exam	Evaluated test/ exam
W_02	Conventional lecture	Test / Exam	Evaluated test/ exam
W_03	Conventional lecture	Test / Exam	Evaluated test/ exam
W_04	Conventional lecture	Test / Exam	Evaluated test/ exam
UMIĘTNOŚCI			
U_01	Practical classes	Test	Evaluated test
U_02	Practical classes	Test	Evaluated test
U_03	Practical classes	Test	Evaluated test
	Practical classes	Test	Evaluated test
KOMPETENCJE SPOŁECZNE			

K_01	Problem-Based Learning	Test	Evaluated test
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VI. Kryteria oceny, wagi.

CLASSES:

At least 80% of attendance is required. Two tests together constitute the final grade:

91 – 100% excellent

81 – 90% very good

71 – 80% good

61 – 70% satisfactory

51 – 60% sufficient

less than 51% fail

LECTURE:

The written exam consists of two parts: practical (60%) - verifying the ability to apply the knowledge in practice, theoretical (40%) - checking theoretical knowledge.

Detailed criteria are given to students with each edition of the subject.

VII. Obciążenie pracą studenta

Forma aktywności studenta	Liczba godzin
Liczba godzin kontaktowych z nauczycielem	135
Liczba godzin indywidualnej pracy studenta	120

VIII. Literatura

Literatura podstawowa
A. Borowkow, Rachunek prawdopodobieństwa, PWN 1977 J. Jakubowski, R. Sztencel, Wstęp do teorii prawdopodobieństwa, Script 2002 P. Billingsley, Prawdopodobieństwo i miara, PWN 1987 W. Feller, Wstęp do rachunku prawdopodobieństwa, t. I-II, PWN 1969 M. Loève, Probability Theory, Van Nostrand 1960 M. Fisz, Rachunek prawdopodobieństwa i statystyka matematyczna, PWN 1967
Literatura uzupełniająca
W. Krysicki i in. – Rachunek prawdopodobieństwa i statystyka matematyczna w zadaniach, t. I-II, PWN 1997