Course Syllabus

I. General Information

Course name	Probability theory
Programme	Mathematics
Level of studies (BA, BSc, MA, MSc, long-cycle MA)	BA
Form of studies (full-time, part-time)	full-time
Discipline	Mathematics
Language of instruction	english

Course coordinator/person responsible	Dr hab. August Zapała

Type of class *(use only the types mentioned below)*	Number of teaching hours	Semester	ECTS Points
lecture	60	V	8
tutorial			
classes	30	V	
laboratory classes			
workshops			
seminar			
introductory seminar			
foreign language classes			
practical placement			
field work			
diploma laboratory			
translation classes			
study visit			

Course pre-requisites	Introduction to mathematics, Introduction to differential and integral calculus, Mathematical analysis I and II (sequences and series of numbers, differential and integral calculus of functions of one variable and many variables) Topology of metric spaces (foundations of set theory, properties of compact metric spaces)

II. Course Objectives

Understanding of methods for mathematical description of random phenomena
Learning how to calculate probabilities of random events, determine distributions of random variables and evaluate numerical parameters of probability distributions
Understanding of various modes of convergence of sequences of random variables
Understanding of characteristic functions (Fourier transformation)

Understanding of basic limit theorems of probability theory

III. Course learning outcomes with reference to programme learning outcomes

Symbol	Description of course learning outcome	Reference to programme learning outcome
	KNOWLEDGE	
W_01	Students familiarize with various definitions of probability, the notions of a random variable, distribution function and probability density	K_W03, K_W01
W_02	Students get acquainted with the most important discrete and continuous probability distributions	K_W03, K_W05
W_03	Students get acquainted with characteristic functions (Fourier transformation) and inversion formulas	K_W04, K_W02, K_W05
W_04	Students get acquainted with the most important limit theorems of probability theory	K_W04, K_W02, K_W05
	SKILLS	
U_01	Students are capable to build and analyse models of random phenomena by means of appropriate probability spaces and evaluate probabilities of random events	K_U30, K_U01, K_U02,K_U04, K_U05, K_U06, K_U29,
U_02	Students can apply the total probability formula and Bayes' formula	K_U32, K_U03
U_03	Students are capable to give various examples of discrete and continuous probability distributions and they know practical applications of these distributions	K_U31, K_U29,
U_04	Students can determine parameters of distributions of discrete and continuous random variables, find equations of regression lines, calculate characteristic functions, and apply limit theorems and laws of large numbers to estimate probabilities	K_U33, K_U35,
	SOCIAL COMPETENCIES	
K_01	Students precisely formulate questions to deepen the understanding of the subject and complement the missing elements of reasoning	K_K01, K_K05

IV. Course Content

The sample space and random events, fields and σ-fields of events. Classical, geometrical and statistical definitions of probability, examples of applications. Axioms of probability., finitely additive probability and axiom of continuity. Independence of events, fields and σ-fields of events. Conditional probability, the law of total probability and Bayes' formula. Discrete probability spaces. Distribution functions in 1- dimensional and multidimensional Euclidean space. Construction of probability measures from distribution functions in 1-dimensional and multidimensional Euclidean space. Random variable, the law and distribution function of the random variable. Discrete and continuous distributions, probability mass function and probability density. Information concerning the Jordan theorem and the Lebesgue-Radon-Nikodym theorem on decomposition of distribution functions. Random vectors. Multidimensional discrete and continuous distributions, probability densities in multidimensional spaces. Marginal distributions of discrete and continuous random vectors. Independent random variables, criteria of

independence for discrete and continuous random variables. Numerical characteristics of random variables. Expectation and its properties. Variance, standard deviation, and their properties. Moments and central moments. Covariance and correlation coefficient, properties of the correlation coefficient. Lines of regression. Various modes of convergence of random variables (in distribution, in probability, almost sure and in p-th mean). A critrion for almost sure convergence. Markov's and Chebyshev's inequalities and certain their applications. Relationships between various modes of convergence. Complex random variables, independence and expectations of complex random variables. Some useful inequalities for complex random variables. Characteristic functions and their properties. Lévy's theorem (the inversion formula for distribution functions). Inversion formulas for discrete distributions. Inversion formulas for probability densities. Helly's lemmas and the Helly-Bray theorem. The Lévy-Cramér theorem. The Lindeberg-Feller central limit theorem for triangular arrays of random variables. The Lindeberg-Feller central limit theorem for a sequence of random variables, Lyapunov's and Lindeberg-Lévy theorems. Weak law of large numbers, Khintchine's, Chebyshev's and Markov's theorems. Theorem on almost uniform convergence of a sequence of characteristic functions. Inequalities for truncated random variables. The notion of median and Levy's symmetrization inequality. Classical criterion for convergence to a constant. The Borel zero-one law and the Borel-Cantelli lemma. Kolmogorov's inequality, Kolmogorov's criterion and the Kolmogorov strong law of large numbers.

V. Didactic methods used and forms of assessment of learning outcomes

Symbol	Didactic methods *(choose from the list)*	Forms of assessment *(choose from the list)*	Documentation type *(choose from the list)*
KNOWLEDGE			
W_01	Conventional lecture	Test or Exam	Evaluated test or Protocol
W_02	Conventional lecture	Test or Exam	Evaluated test or Protocol
W_03	Conventional lecture	Test or Exam	Evaluated test or Protocol
W_04	Conventional lecture	Test or Exam	Evaluated test or Protocol
SKILLS			
U_01	Practical classes	Written test	Evaluated test
U_02	Practical classes	Written test	Evaluated test
U_03	Practical classes	Written test	Evaluated test
SOCIAL COMPETENCIES			
K_01	Problem-Based Learning	Test	Evaluated test

VI. Grading criteria, weighting factors.....

CLASSES: At least 80% of attendance is required. Two tests together constitute the final grade: 91 – 100% excellent 81 – 90% very good 71 – 80% good 61 – 70% satisfactory 51 – 60% sufficient less than 51% fail

LECTURE: The written exam consists of two parts: practical (60%) - verifying the ability to apply the knowledge in practice, theoretical (40%) - checking theoretical knowledge. Detailed criteria are given to students with each edition of the subject.

VII. Student workload

Form of activity	Number of hours
Number of contact hours (with the teacher)	**90+30(consultations)**
Number of hours of individual student work	**120**

VIII. Literature

Basic literature
A. Borovkov, Probability Theory,Springer-Verlag, London 2013, PWN 1977 (Polish ed.) M. Loève, Probability Theory, Van Nostrand 1960 P. Billingsley, Probability and Measure, PWN 1987 (Polish ed.) J. Jakubowski, R. Sztencel, Wstęp do teorii prawdopodobieństwa, Script 2002 W. Feller, An Introduction to Probability Theory and Its Applications, t. I–II, PWN 1969 (Polish ed.) M. Fisz, Rachunek prawdopodobieństwa i statystyka matematyczna, PWN 1967
Additional literature
W. Krysicki i in. Rachunek prawdopodobieństwa i statystyka matematyczna w zadaniach, t. I-II, PWN 1997