Course Syllabus

I. General Information

| Course name | Complex analysis |
|--|------------------|
| Programme | Mathematics |
| Level of studies (BA, BSc, MA, MSc, long-cycle | BA |
| MA) | |
| Form of studies (full-time, part-time) | Full-time |
| Discipline | Mathematics |
| Language of instruction | english |

| Course coordinator | Dr hab. Dariusz Partyka |
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| | |

| Type of class (use only the types mentioned below) | Number of teaching hours | Semester | ECTS Points |
|--|--------------------------|----------|-------------|
| lecture | 30 | V | 5 |
| tutorial | | | |
| classes | 30 | V | |
| laboratory classes | | | |
| workshops | | | |
| seminar | | | |
| introductory seminar | | | |
| foreign language | | | |
| classes | | | |
| practical placement | | | |
| field work | | | |
| diploma laboratory | | | |
| translation classes | | | |
| study visit | | | |

| Course pre-requisites | Basic knowledge of mathematical logic and set theory, mathematical | |
|-----------------------|--|--|
| | analysis, topology and analytic geometry. | |

II. Course Objectives

- C1. Presentation of basic concepts and facts of complex analysis.
- C2. Introduction students to the theory of holomorphic functions.
- C3. Familiarize students with applications of the theory of holomorphic functions in other areas of mathematics and natural sciences like e.g. real analysis.

III. Course learning outcomes with reference to programme learning outcomes

| Symbol | Description of course learning outcome | Reference to programme learning outcome |
|-----------|--|---|
| KNOWLEDGE | | |
| W_01 | /_01 The student understands what is the process of proving in K_W01, K_W02, | |

| | mathematics. Understands the importance of counterexamples in mathematical reasoning. | K_W05 |
|------|---|--|
| W_02 | The student knows the basic concepts of complex analysis and the basic properties of the differential calculus of complex functions of one complex variable. Is familiar with examples illustrating these concepts and properties. Knows the proofs of the fundamental theorems in this scope. | K_W02, K_W03, K_W04,K_W05 |
| W_03 | The student is familiar with the basics of the theory of holomorphic functions of one complex variable. | K_W02, K_W04, K_W05 |
| | SKILLS | |
| U_01 | The student uses the properties of the field of complex numbers. Fluently performs algebraic operations on complex numbers. Can operate on geometric and topological concepts of the complex plane and the extended complex plane. Can use their basic properties. He can use the geometric interpretation of complex numbers. | K_U01, K_U02, K_U03, K_U04, K_U05, K_U06 |
| U_02 | The student is able to apply theorems and methods of real analysis in the case of complex functions. Calculates the limits of complex functions. Can examine the summation of complex sequences, in particular power series and Laurent's series. | K_U01, K_U02, K_U03, K_U04, K_U06 |
| | Can investigate the continuity and differentiability of complex functions. Is able to determine derivative and primitive functions of holomorphic ones. | |
| U_03 | The student uses the concept of the curvilinear integral of a complex function. Can calculate such integrals for holomorphic functions using Cauchy's integral theorem, Cauchy's integral formula and theorems about the existence of a primitive function. Can determine the index of a point with respect to a curve by using its properties. Can expand the holomorphic functions into power series and Laurent's series. Classifies isolated singularities of holomorphic functions. Applies the residue theorem in order to determine the curvilinear integrals of holomorphic functions. Is able to use the argument principle and Rouché's theorem in order to localize the zeros and poles of holomorphic functions. Can calculate sums of some complex sequences using residues. | K_U01, K_U02, K_U04, K_U06 |
| | SOCIAL COMPETENCIES | |
| K_01 | Student understands the need to further develop his knowledge and skills in algebra. Can formulate questions in order to better understand the subject. | K_K01 |
| K_02 | The student can present issues dealing with algebra in an understandable way. | K_K05 |

IV. Course Content

- 1. The field of complex numbers.
- 2. The complex plane and the extended complex plane.
- 3. The limit and continuity of complex functions.
- 4. Convergence and summation of complex sequences.

- 5. Differentiability of complex functions in the real and complex sense.
- 6. Holomorphic functions. Polynomials and rational functions.
- 7. Power series.
- 8. Differentiability of power series.
- 9. Primitive functions of complex functions.
- 10. The exponencial function and its properties.
- 11. Fundamental elementary functions.
- 12. Riemann-Stieltjes integral of a complex function. Curvilinear integrals.
- 13. Goursat's lemma and its generalizations.
- 14. Integrating holomorphic functions in star-like domains. The special case of Cauchy's integral theorem.
- 15. Index of a point with respect to a curve.
- 16. Cauchy's integral formula for holomorphic functions in star-like domains.
- 17. Liouville's Theorem and the fundamental theorem of algebra.
- 18. Expansions of holomorphic functions into power series.
- 19. Roots of holomorphic functions.
- 20. Isolated singular points of holomorphic functions. Meromorphic functions.
- 21. The residue theorem for meromorphic functions in star-like domains.
- 22. The argument principle and Rouché's theorem for meromorphic functions in star-like domains.
- 23. The open mapping theorem for holomorphic functions.
- 24. The maximum principle for holomorphic functions.
- 25. Summing complex sequences using residues.
- 26. Laurent's series.

V. Didactic methods used and forms of assessment of learning outcomes

| Symbol | Didactic methods (choose from the list) | Forms of assessment (choose from the list) | Documentation type (choose from the list) |
|---------------------|--|--|---|
| | KNOWLEDGE | | |
| W_01 | Conventional lecture. | Exam, oral test. | Written test. |
| W_02 | Conventional lecture. | Exam, oral test. | Written test. |
| W_03 | Conventional lecture. | Exam, oral test. | Written test. |
| SKILLS | | | |
| U_01 | Practical classes. | Test. | Evaluated test. |
| U_02 | Practical classes. | Test. | Evaluated test. |
| U_03 | Practical classes. | Test. | Evaluated test. |
| SOCIAL COMPETENCIES | | | |
| K_01 | Discussion. | Observation. | Rating card. |
| K_02 | Discussion. | Observation. | Rating card. |

VI. Grading criteria, weighting factors.....

LECTURE:

The completion of classes is required.

Written and oral exam together constitute the final grade:

- 91 100% (5,0)
- 81 90% (4,5)
- 71 80% (4,0)
- 61 70% (3.5)
- 51 60% (3,0)
- 0 50% (2,0)

CLASSES:

At least 80% of attendance is required.

Two tests together constitute the final grade:

- 91 100% (5,0)
- 81 90% (4,5)
- 71 80% (4,0)
- 61 70% (3,5)
- 51 60% (3,0)
- 0 50% (2,0)

Detailed rules of evaluation are given on lectures and classes.

VII. Student workload

| Form of activity | Number of hours |
|--|-----------------|
| Number of contact hours (with the teacher) | 90 |
| Number of hours of individual student work | 60 |

VIII. Literature

Basic literature

Lecture notes and lecture notes in electronic form as well as:

- 1. J.B. Conway, Functions of One Complex Variable, Springer, New York 2012.
- 2. W. Rudin, Real and Complex Analysis, 3 ed., McGraw-Hill, 1987.
- 3. J. Chądzyński, Wstęp do analizy zespolonej, PWN, Warszawa 1999.
- 4. J. Krzyż i J. Ławrynowicz, Elementy analizy zespolonej, WNT, Warszawa 1981.
- 5. F. Leja, Funkcje zespolone, PWN, Warszawa 1979.

Additional literature

- 1. L.V. Ahlfors, Complex Analysis, 3 ed., McGraw-Hill, 1979.
- 2. J.G. Krzyż, Problems in Complex Variable Theory, Elsevier, New York 1972.
- 3. S. Saks i A. Zygmund, Funkcje analityczne, PWN, Warszawa 1959.
- 4. B.W. Szabat, Wstęp do analizy zespolonej, PWN, Warszawa 1974.
- 5. E. Kącki i L. Siewierski, Wybrane działy matematyki wyższej z ćwiczeniami, PWN, Warszawa 1979.