An Introduction to Multilevel Analysis

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Once you know that
hierarchies exist,
you see them everywhere

Kreft and deLeeuw (1998)

Basic Multilevel Data Structure

- Observations at one level of analysis are nested within observations at another
- Persons within groups such as work groups, classrooms, clinics, families, and so forth
- Observations within persons -- various types of diary studies

Multilevel Analyses

- Sometimes such data sets are referred to as "nested" or "hierarchically nested" because observations (also referred to as units of analysis) at one level of analysis are nested within observations at another level.
- This nesting needs to be taken into account when you analyze the data. If you do not, you violate important assumptions about independence of errors.

Independence of errors

- For example, all the children in the same classroom have the same teacher. Although the students are individuals, they share a teacher. This means that they cannot be treated as independent observations
- Students in another class are also individuals and have the same teacher, but this teacher is different than the teacher in class #1.

Independence of errors

- When analyzing student level measures (e.g., reading and studying) the fact that students in the same class have the same teacher but have a different teacher than students in another class needs to be controlled.
- Multilevel modeling does this in the most accurate way that is currently available.

Advantages of Multilevel Analyses

- Parameter estimates incorporate effects of hierarchies
- Analyze phenomena (means, variances and covariances) at multiple levels simultaneously
- Very important: Relationships (covariances) can differ across levels of analysis

Relationships Across Different Levels of Analysis

- Relationships can vary across different levels of analysis
- Relationships at different levels of analysis are mathematically independent
- Knowing the relationship at one level of analysis tells us nothing about relationships at the another level of analysis

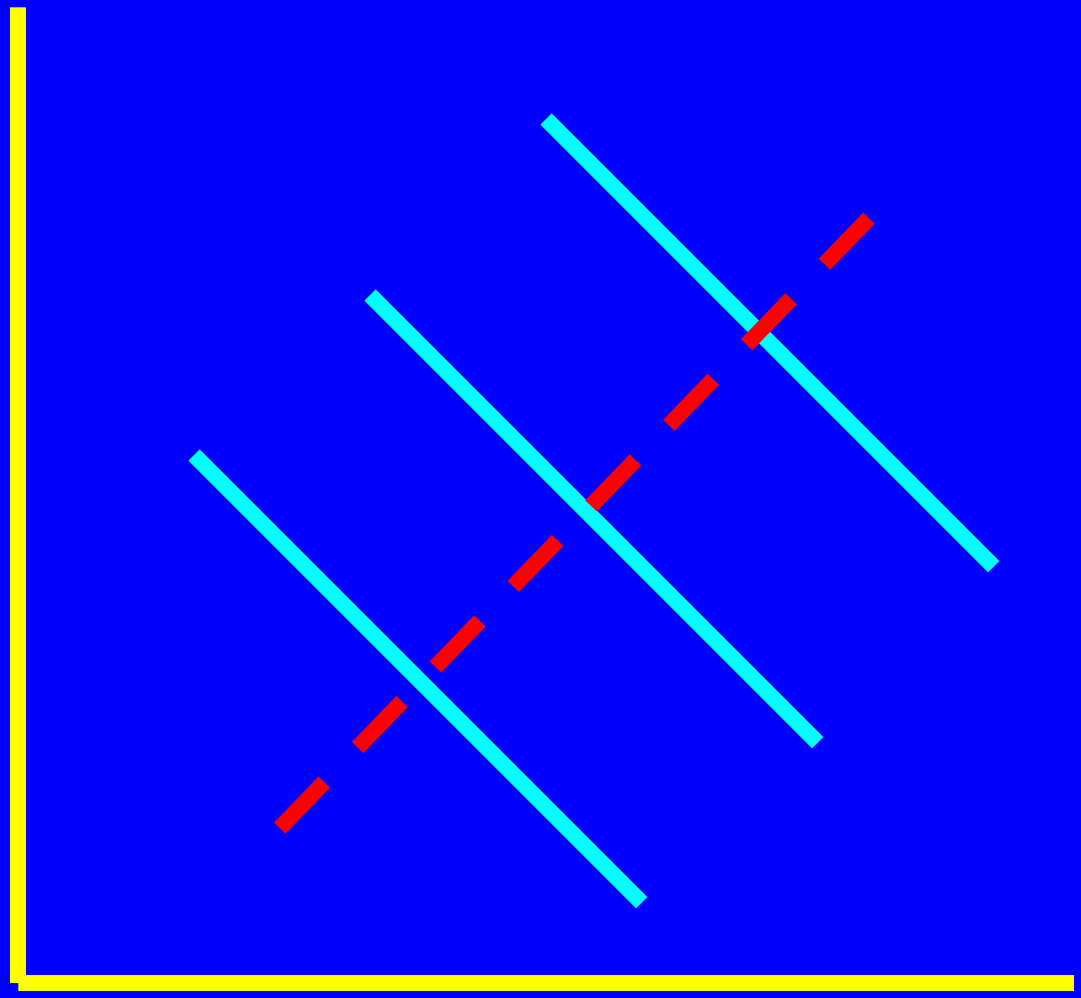
An Example: Relationships between Productivity and Group Identity

- Three groups with 5 people in each group.
- For each person, we measure
  - Productivity -- how much the person produces
  - Identity -- how much does the person identify with the group

Negative Within-Group
Positive Between-Group

Group 1		Group 2		Group 3	
1	8	4	13	9	18
2	7	5	12	10	17
3	6	6	11	11	16
4	5	7	10	12	15
5	4	8	9	13	14
3	6	6	11	11	16

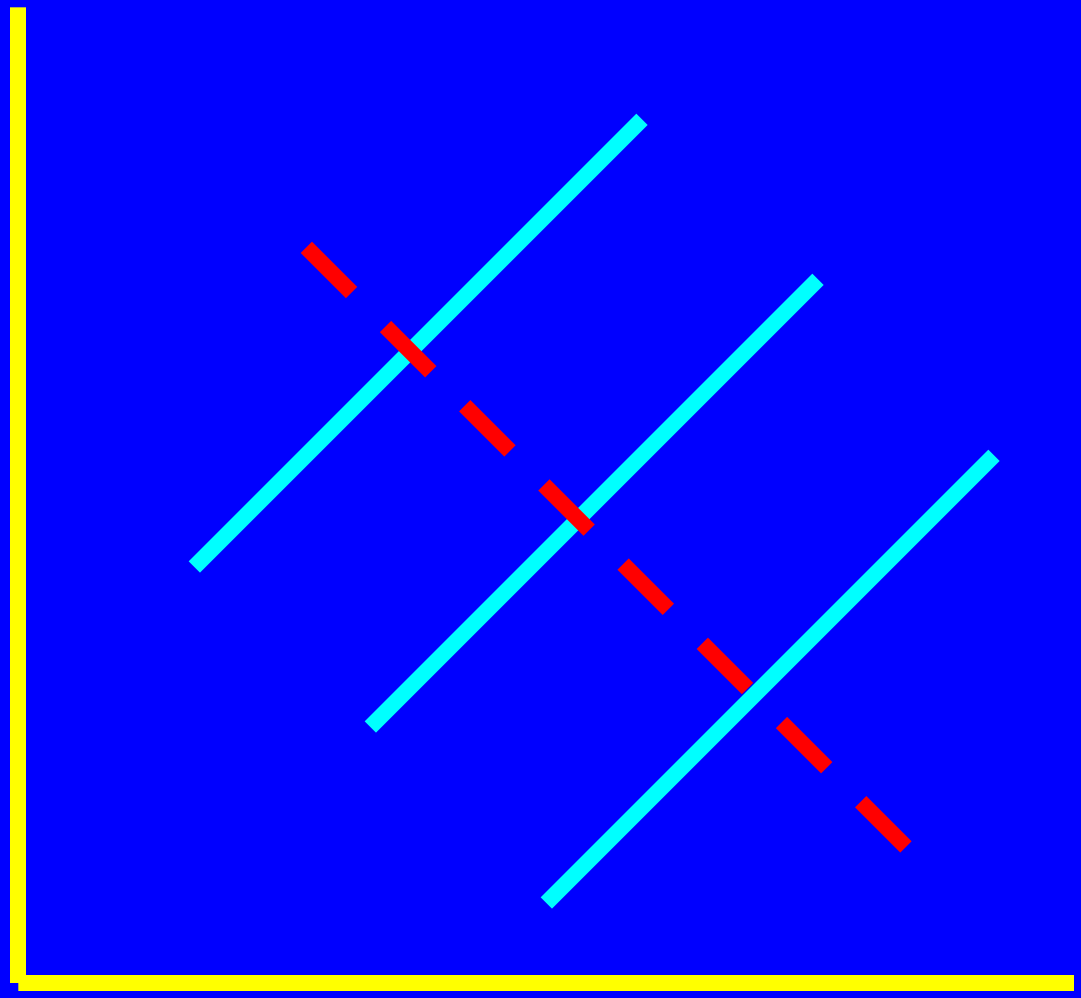
Negative Within-Group
Positive Between-Group



Positive Within-Group
Negative Between-Group

Group 1		Group 2		Group 3	
6	11	9	9	11	6
7	12	10	10	12	7
8	13	11	11	13	8
9	14	12	12	14	9
10	15	13	13	15	10
8	13	11	11	13	8

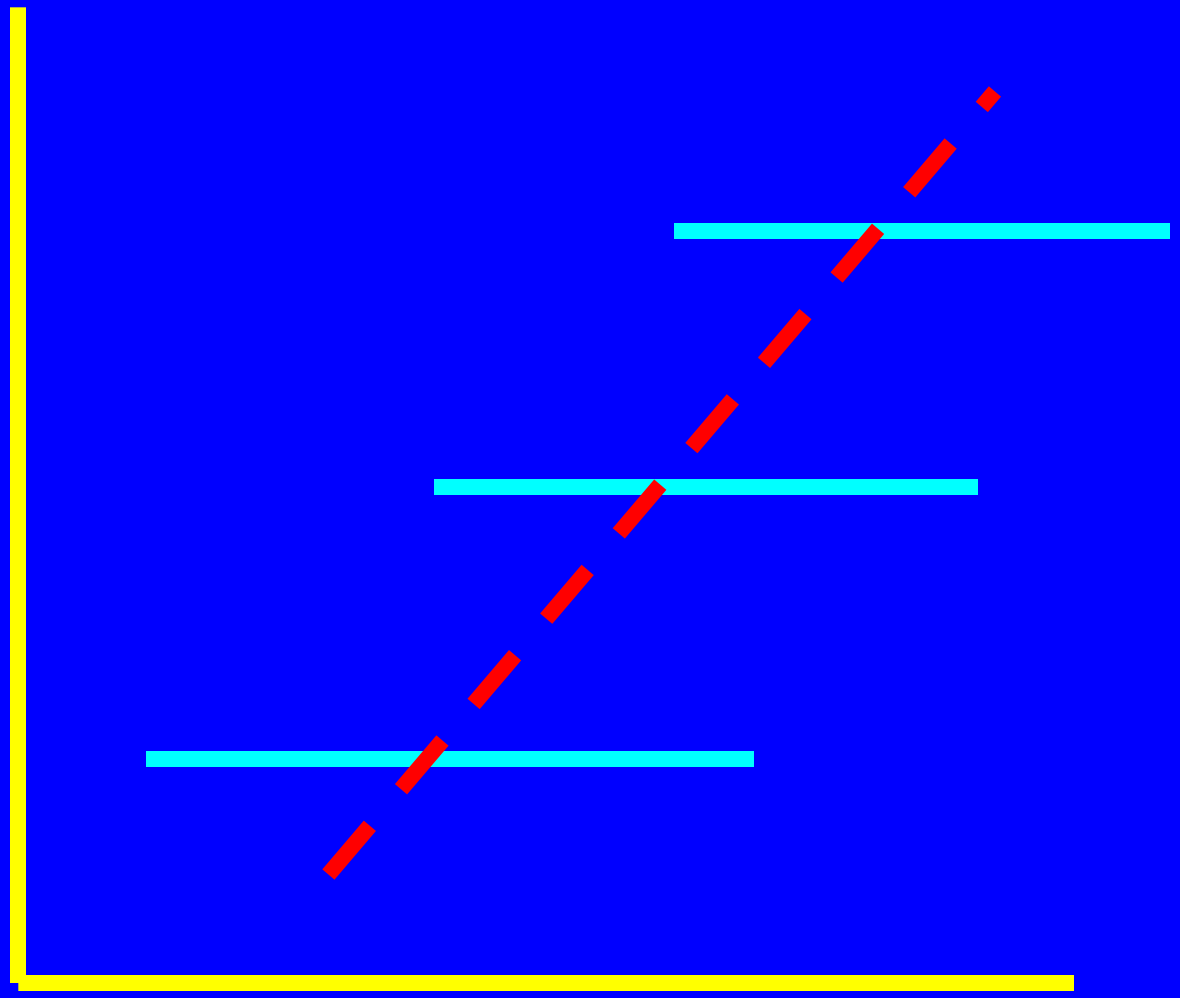
Positive Within-Group
Negative Between-Group



No Within-Group
Positive Between-Group

Group 1		Group 2		Group 3	
1	8	4	10	9	15
2	8	5	10	10	15
3	8	6	10	11	15
4	8	7	10	12	15
5	8	8	10	13	15
3	8	6	10	11	15

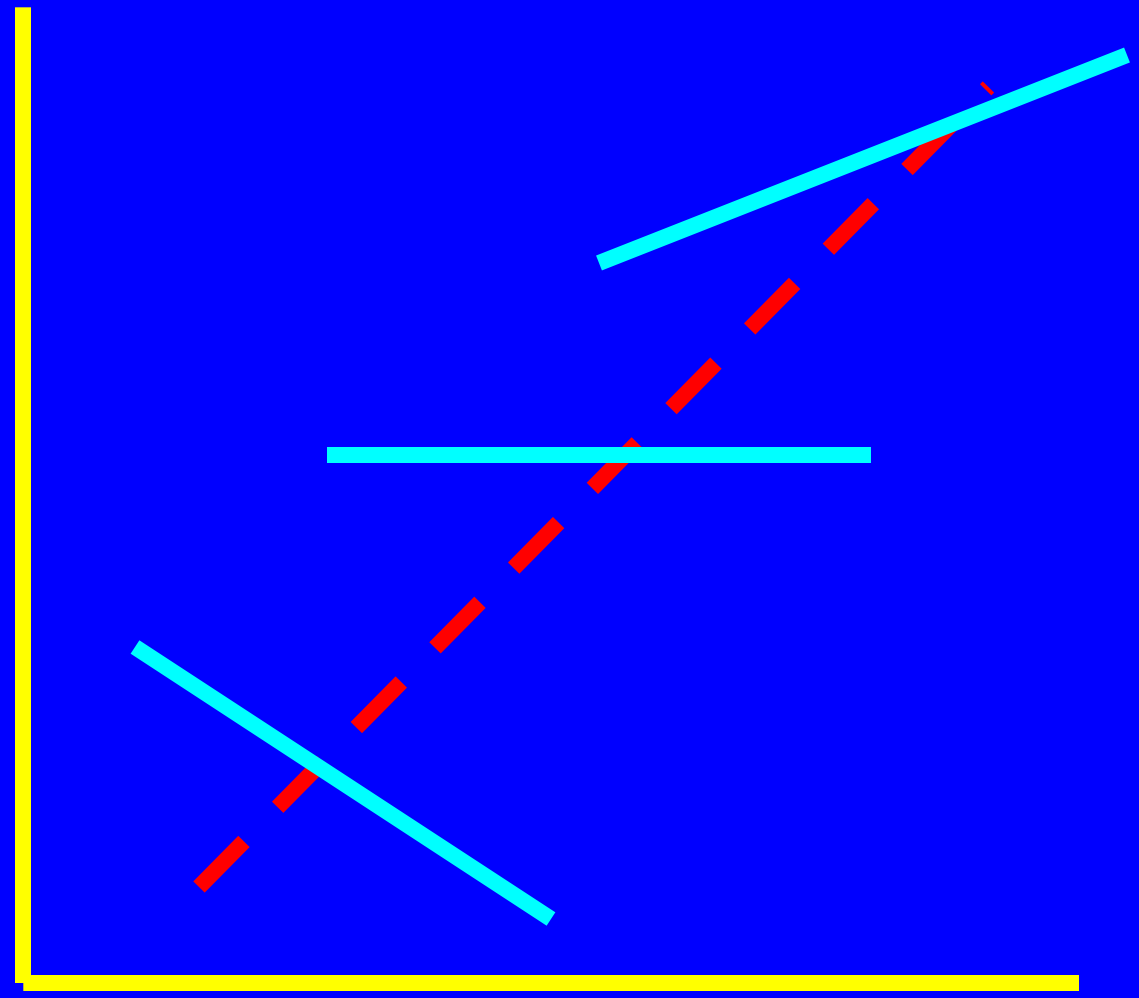
No Within-Group
Positive Between-Group



Varying Within-Group Positive Between-Group

Group 1		Group 2		Group 3	
1	10	4	10	9	13
2	9	5	10	10	14
3	8	6	10	11	15
4	7	7	10	12	16
5	6	8	10	13	17
3	8	6	10	11	15

Varying Within-Group Positive Between-Group



Review of other methods

- Other ways of analyzing multilevel data structures.
- Although useful in some ways, the methods described next are not as accurate as multilevel random coefficient modeling.
- Accuracy defined in terms of Monte Carlo studies comparing different techniques.

Ordinary Least Squares Multilevel Analyses

- Aggregated means analyses
- Dummy-coded least squares (LSDV)
- Sub-group analyses
  - Regression by groups
  - Group level correlations

Aggregated means

- Calculate within-group means
- Examine relationships between/among these means
- Correlations, ANOVAS, regression, etc.
- Advantages: Simple, familiar
- Disadvantages: Ignores within-group relationships, inaccurate parameter estimates

LSDV Analysis
Least-Squares Dummy Variable

- Add dummy variables representing group membership, e.g., five groups
  - y = intercept + g1 + g2 + g3 + g4 + x
- Advantages: Familiar (OLS regression), takes into account between-group differences in means
- Disadvantages: Need g-1 variables, does not examine how x-y relationship varies across groups (Interaction terms can be included). Does not model error properly.

Sub-group analyses

- Analyze within-group relationships
- Regression by group analysis
  - Calculate regression equations for each group
  - Compare equations with F-test (g-1 df)
- Calculate within-group correlations (or regression coefficient) and use these as dependent measures

Sub-group analyses

- Advantages: Can analyze within-group relationships and how they vary as a function of other group-level measures
- Disadvantages: Does not account for differences among groups in reliability of estimates (group size and consistency of responses), does not model error properly (treats parameters as fixed, not random)

When to use Multilevel Modeling

- When the data are nested
- Ignore advice about ICC (intraclass correlations)
- ICCs concern distribution of means only
- Interest usually focuses on relationships
- There may be no between group variance in two measures, but there may still be between group variability in relationships

Problems of Relying on ICCs to make Decisions about Multilevel Analyses

- In the next example, the ICCs are all 0. All means are 3. There is no between-group variability for either measure.
- Yet, the relationship between x and y varies across the 6 groups
- Positive in 1, 2, 3, negative 4, 5, 6.

Group 1		Group 2		Group 3	
x	y	x	y	x	y
1	5	1	5	1	5
2	4	2	4	2	4
3	3	3	3	3	3
4	2	4	2	4	2
5	1	5	1	5	1
Group 4		Group 5		Group 6	
x	y	x	y	x	y
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5

Multiple levels of random sampling

- Traditionally, people (observations) are the random element -- sampled from population
- In multilevel structures, organizing units may also be sampled from populations
- Classes randomly sampled from schools
- Work groups randomly sampled from factory

Logic of Multilevel Modeling

- Coefficients describing level 1 phenomena are estimated for each level 2 unit
  - Intercepts--means
  - Slopes--covariances (like regression coef.)
- Level 1 coefficients are then analyzed at level 2

Typical Data Structure

- Individuals nested with groups
  - Students in classrooms
  - Worker in work groups

Level 1 Equations: Intercepts Means for Each Group

- Intercept (mean productivity)
  - $y_{ij} = \beta_{0j} + r_{ij}$
- y productivity
  - i individuals
  - across j groups
  - r individual level error

Totally unconditional model

- Level 1 model: $y_{ij} = \beta_{0j} + r_{ij}$
- Level 2 model: $\beta_{0j} = \gamma_{00} + \mu_{0j}$
- "Totally unconditional" because no predictors at either level
- Provides estimates of variances at each level (within-group, r, between group μ) which suggest where analyses with predictors might be productive

Level 2 Equations: Intercepts Do Group Means Differ?

- Mean productivity for each group
  - $\beta_{0j} = \gamma_{00} + \mu_{0j}$
- Mean productivity and a group level variable (time together)
  - $\beta_{0j} = \gamma_{00} + \gamma_{01}$ (Time) $+ \mu_{0j}$

Level 1 Slope: Within-group relationship between Productivity & Identification

- Relationship between productivity and identification
  - $y_{ij} = \beta_{0j} + \beta_{1j}$ (Identification) + r_{ij}
- Intercept for each group β_{0j}
- Identification slope for each group β_{1j}

Level 2 Equations: Slopes Does P-I Relationship Vary?

- Mean productivity-identification relationship across all groups
  - $\beta_{1j} = \gamma_{10} + \mu_{1j}$
- Does productivity-identification relationship vary as a function of how long the group has been together?
  - $\beta_{1j} = \gamma_{10} + \gamma_{11}(\text{Time}) + \mu_{1j}$

Analyses of slopes

- Are sometimes called “slopes as outcomes” analyses because the slope from level 1 becomes the dependent measure at level 2
- or… “cross level interactions” because relationships at level 1 vary as a function of a variable at level 2
- or… moderation analyses because a variable at level 2 moderates a relationship at level 1

Analyses of slopes

- Provide much more accurate estimates of differences in covariances (correlations) than other methods.
- Can also analyze within-person correlations -- difficult to do using OLS.

In conclusion

- Multilevel modeling (MLM) can answer different and more sophisticated questions than comparable OLS analyses and do so more accurately.
- MLM is being used more frequently in many disciplines.

Some helpful references

- Kreft, I. G. G., & de Leeuw, J. (1998). *Introducing multilevel modeling*. Newbury Park, CA: Sage Publications.
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