# Emergence and Effective Theory of the Universe – the Case Study of Lambda Cold Dark Matter Cosmological Model

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**Abstract**

Recent astronomical observations strongly indicate that the current Universe is undergoing an accelerated phase of its expansion. If the Universe evolution is described by the FRW model then the acceleration should by driven by some perfect fluid substance violating the strong energy conditions. Hence the negative pressure is required for the explanation of acceleration. While different candidates for such a fluid termed dark energy are suggested, the simple candidates for dark energy in the form of positive cosmological constant seems to be the best one. However there is no simple physical interpretation of the Lambda term as a quantum vacuum energy because of the fine tuning problem. We argue in the paper that the LCDM model as well as the CDM one has status of effective theory only. The paper is concerned with the significance of this approach to the description of the Universe. Especially we study the methodological status of the cosmological models from the point of view of the debate on reductionism end emergence in the science. We pointed out that modern effective cosmological theories may provide an interesting case study in the current philosophical discussion. We also advocated that notion of structural stability may by useful in our understanding of the relations of the emergence and reduction between cosmological models. The structural stability of the LCDM model can be interpreted as a property of flexibility of the model to accommodate the observational data. Therefore one can explain why the LCDM model is the best one in confrontation of dark energy cosmology with observations.

## 1 Introduction

Recent astronomical observations of distant supernovas SNIa type strongly indicate that the current Universe is undergoing an accelerated phase of expansion [1, 2, 3, 4, 5, 6]. If the Universe evolution is described by homogeneous and isotropic models filled with perfect fluid then the acceleration should by driven by a perfect fluid violating the strong energy condition. While different candidates for such a fluid termed dark energy are

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suggested, the simple candidates for the dark energy in the form of positive cosmological constant seems to be the best one [7, 8]. While the Lambda CDM model is a good phenomenological description of the acceleration phase of the expansion of the Universe there is serious problem with the interpretation of the Lambda term as a quantum vacuum energy because of the fine tuning problem [9, 10]. We argue that while the LCDM has the status of an effective theory which offers description of the observational facts (rather than their explanation) this theory introduce principally the new theoretical element which plays the role of an effective parameter changing dramatically the dynamics.

The theory which is called an effective theory (although it is not yet a technical notion) is characterized by a few important features:

- An effective theory *works* in a certain field of physics. In most cases this scope of application is described in terms of energy or distance scale. The theory which is "effective" in a specific physical regime describes "behavior" of elaborated objects but often does not explain the nature of them. For example the standard model is the effective theory of gluons and quarks in the distant scale of $10^{-17}$ m. Intuitively that feature of effective theories was described by H. Georgi [11]:

  > (...)We can divide up parameter space of the world into different regions, in each of which there is a different appropriate description of the important physics. Such an appropriate description of the important physics is an "effective theory". The two key words here are appropriate and important.
  >
  > The word "important" is key because the physical processes that are relevant differ from one place in parameter space to another. The word "appropriate" is key because there is no single description of physics that is useful everywhere in parameter space [11].

- Every effective theory uses parameters which can be called "informational input" being assigned to the theory without explanations; i.e. we do not have to understand the nature of these *input parameters* as to successfully operate the theory. It is important to distinguish *input parameters* from any kind of information being used by the theory or model (initial conditions for example ). Specific parameters do have the status of *input parameters* only in the frame of the effective theory[1]. Their values can be determined experimentally but only the more fundamental theory in fact provide explanation for them being like that.

- Therefore, we can say for example that the nucleus spin, the elementary charge or the magnetic property are *input parameters* for the effective theory which uses them successfully but without understanding nature of them. The effective theories can be put into specific series with respect to *the input parameters.* This is called by some philosophers of science a tower of effective theories [13].

- We have written that an effective theory could be used in a certain area of physics. Indeed this kind of theory *works* successfully ("effectively") on that level but it breaks

[1] "For instance, temperature is an example of a feature that is relevant in thermodynamics but irrelevant in Newtonian or statistical mechanics. Light rays are relevant in geometric optics, but they are irrelevant in Maxwell's electrodynamics. The chirality of molecules is relevant in physical chemistry , but it is irrelevant in a Schrödinger-type quantum mechanical description. Nevertheless, there are strategies for implementing the context due to which temperature is relevant in thermodynamics, due to which rays are relevant in geometric optics, a due to which chirality is relevant in physical chemistry, at the level of statistical mechanics, of electrodynamics, and of quantum mechanics respectively". [12, p.1760]

down when we exceed the limits of its application. The very important feature of that limited applicability criterium is that these regimes have to be easily separated.

- The effective theories coexist with each other. Let's consider two cases. We presented in Table I a few examples of coexisting (but independent) different dark energy models. In this case one can speak of sets of models. If two or more theories or models are used to solve current physical problem (for example the problem of accelerating Universe) we can evaluate their *effectiveness* by pointing out what type of knowledge they provide. It is also possible to explain relations between effective theories in terms of structures. An effective theory is not a fundamental theory. We called that kind of structure of effective theories – the series or tower of theories. It is possible because the theories of that kind not only coexist with each other (but not in the same regime) but also condition each other. The theory on the lower level determines some parameters for the theory of upper one. That kind of relation can be elaborated by using the notions of emergence or supervenience [2]. If we study that series of the effective theories we can meet the problem of a fundamental theory, a base–theory. This final theory actually *ends* that series in the terms of a methodological reconstruction, but at the same time *begins* the series in the framework of emergence. One can speculate about the possibility of existence of the fundamental theory, but there are also opinions that searching for the series of effective theories leads to theoretical view described by expression: *never-ending tower of effective theories* [13].

We study methodological status of the concordance cosmological LCDM model from the view point of the debate on reductionism and emergence in science [14, 15]. Our main result is that structural stability notion taken from the dynamical system theory may be useful in our understanding of the emergence CDM to LCDM model as well as in understanding of the reduction LCDM to CDM one. We argue that the concepts of structural stability might be a suitable setup for the philosophy of cosmology discussion.

The LCDM model should be treated in our opinion as an effective theory for the following reasons:

1. Theory of gravity which describe the gravitational sector of cosmology is very complicated but if we postulate some simplified assumption like symmetry assumption idealization then we obtain simplest model which can be representing in the form of the dynamical system. In the cosmology assumption of homogeneity and isotropy of space like sections of constant cosmic time ($t = const$) seems to be justified by the distribution of large scale structure of astronomical objects (cosmological principle). If we assume that the topology of spacetime is $R \times M^3$, where $M^3$ is a maximally symmetric 3-dimensional space then we obtain a geometrical structure of the spacetime modulo a single function of $a(t)$ called the scale factor. In order if we postulate that source of gravity is in the form of perfect fluid with energy density $\varrho(t)$ and pressure $p(t)$ then Einstein field equation reduces to the ordinary system of differential equation determining a single function $a(t)$. These equations called Friedmann equations FRW model describe the evolution of the Universe at the large

[2]Supervenience is the most important concept which appears in every discussion about emergence. In our opinion supervenience is a stronger relation than emergence, because it is synchronic in principle. It means that using supervenience notion it is possible to characterize consecutive emerged levels connected with each other, but without describing the very process of emerging the specific level form the more fundamental one. It can be seen as the defect of the theory, but it (the theory) simply does not aspire to explaining these mechanisms.

scale. Of course this model is very simplified but it can be very useful instrument of construction of a new effective theory of the Universe (heuristic function of the model). Recently many authors (see [16, 17]) argue that observational cosmology will change significantly the essence of our world-view.

2. Basing on this simple toy model one can effectively derive observables (for example cosmological tests) which can be used in testing theory itself (function of testing the theory). Note that this is impossible basing on general relativity without the symmetry assumption where there is universal time conception.

3. The general acceptance of LCDM as the working is good strategy [18, 19] but one may also seek alternative physics (pragmatism). After many years of hypotheses and markets of models we have standard cosmological models which is leading as to joint the physical model of the World.

## 2 Structural stability issues

Einstein field equations constitute in general very complicated system of partial nonlinear differential equations but in the cosmology important role plays its solutions with some symmetry assumptions postulated at the very beginning. Usually the symmetry of homogeneity and homogeneity and isotropy is assumed. In this case Einstein field equations can be reduced to the system of ordinary differential equations, i.e. dynamical systems. Hence to the cosmology could be applied the dynamical system methods in natural way. The application of these methods allows to reveal some stability properties of particular visualized in geometrical way as a trajectories in the phase space. Hence one can see how large is the class of solutions leading to the desired property in tools of the attractors and the inset of limit set (an attractor is a limit set with an open inset – all the initial conditions that end up in the some equilibrium state). The attractors are the most prominent experimentally. It is because the probability of an initial state of the experiment to evolve asymptotically to the limit set it is proportional to the volume of inset.

The idea, now called structural stability emerged early in the history of dynamics investigation in 1930's the writings of Andronov, Leontovich and Pontryagin in Russia (1934) [20] (the authors don't use the name *structural stability* but rather the name "roughly systems"). This idea is based on the observation that an actual state of the system can never be specified exactly and application of the dynamical systems might be useful anyway if it can describe the features of the phase portrait that persist when the state of the system is allowed to move around (see [21] for more comments).

Among all dynamicists there is shared prejudice that:

1. There is a class of phase portraits that are far simpler than arbitrary ones which can explain why a considerable portion of the mathematical physics has been dominated by the search for the generic properties. The exceptional case should not arise very often in application and they de facto interrupt discussion (classification) [21, p.349].

2. The physically realistic models of the world should possess some kind of the structural stability because to have many dramatically different models all agreeing with observation would be fatal for the empirical method of science [22, 23] (see also [24, 25, 26, 27, 28].

These prejudice in the Holton terminology can be treated as a thematic principles [29, 30]. In the cosmology a property (for example acceleration) is believed to be "physically realistic" if it can be attributed the generic subsets of the models within a space of all admissible solutions or if it possesses a certain stability, i.e. if it is shared by a "epsilon perturbed model". For example G. F. R. Ellis [31] to formulate so called a probability principle "The Universe model should be one that is a probable model within the set of all universe models" and a stability assumption which states that "the Universe should be stable to the perturbations". The problem is how to define:

1. the space of state and its equivalence,

2. the perturbation of the system.

The dynamical system is called structurally stable if all $\delta$-perturbation of it (sufficiently small) have the epsilon equivalent phase portrait. Therefore for the conception of structural stability we considered a $\delta$-perturbation of vector field determined by the right-hand sides of the system which is small (measured by delta). We also need a conception of the the epsilon equivalence. This has the form of topological equivalence–a homeomorphism of the state space preserving the arrow of time on each trajectory. In the definition of structural stability considered only the deformation of "rubber sheet" type stretches or slides the phase space a small amount measured by epsilon.

There are developed other concepts of stability used by some authors [32, 33]. For example concepts of rigidity and fragility is used in the sense that the attractor solutions never change as long as some conditions are met. In the structural stability conception the global dynamics is important rather than the fragility of solutions against changes in the shape of a functional form of the Hubble function. It is also used the concept of rigidity in the context of a final theory of physics (TOE). Roughly speaking a mathematical structure is said to be rigid, with respect to a certain deformation parameter, if its every deformation with respect to this parameter yields again the same structure [34, 35] (see also [36]). It is interesting that while the deformation parameter is not defined uniquely, the deformation procedure can be strictly defined. The main advantage of the structural stability is that it is the characterization of global dynamics itself.

Fig. 1 illustrates the property of structural stability of single spiral attractor (focus) and saddle point and structural instability of center. The addition of a delta perturbation pointing outward (no matter how weak) results in a point repeller. We call such a system structurally unstable because the phase portrait of the center and focus are not topologically equivalent (note that all phase curves around the center are closed in contrast to the focus. Hence one can claim that a pendulum system (without friction) is structurally unstable.

Idea of structural stability attempts to define the notion of stability of differential deterministic models of the physical processes. In the case of planar dynamical systems (as in the case of models under consideration) there is true Peixoto theorem (Peixoto 1982) [37] which states that structurally stable dynamical systems form open and dense subsets in the space of all dynamical systems defined on the compact manifold. This theorem is basic characterization of structurally stable dynamical systems in the plane which offers the possibility of exact definition generic (typical) and nongeneric (exceptional) cases (properties) in tools of the notion of structural stability. Unfortunately there is no counterparts of this theorem in more dimensional case when structurally unstable systems can also form open and dense subsets. For our aims, it is important that Peixoto theorem can give the characterization of generic cosmological models in terms of potential function $V$ of the scale factor $a$ which determine the motion of the system of Newtonian type: $\ddot{a} = -\frac{\partial V}{\partial a}$.

Therefore we can treat FRW equation with various forms of dark energy as the two-dynamical systems which looks like Newtonian type where the role of coordinate variable

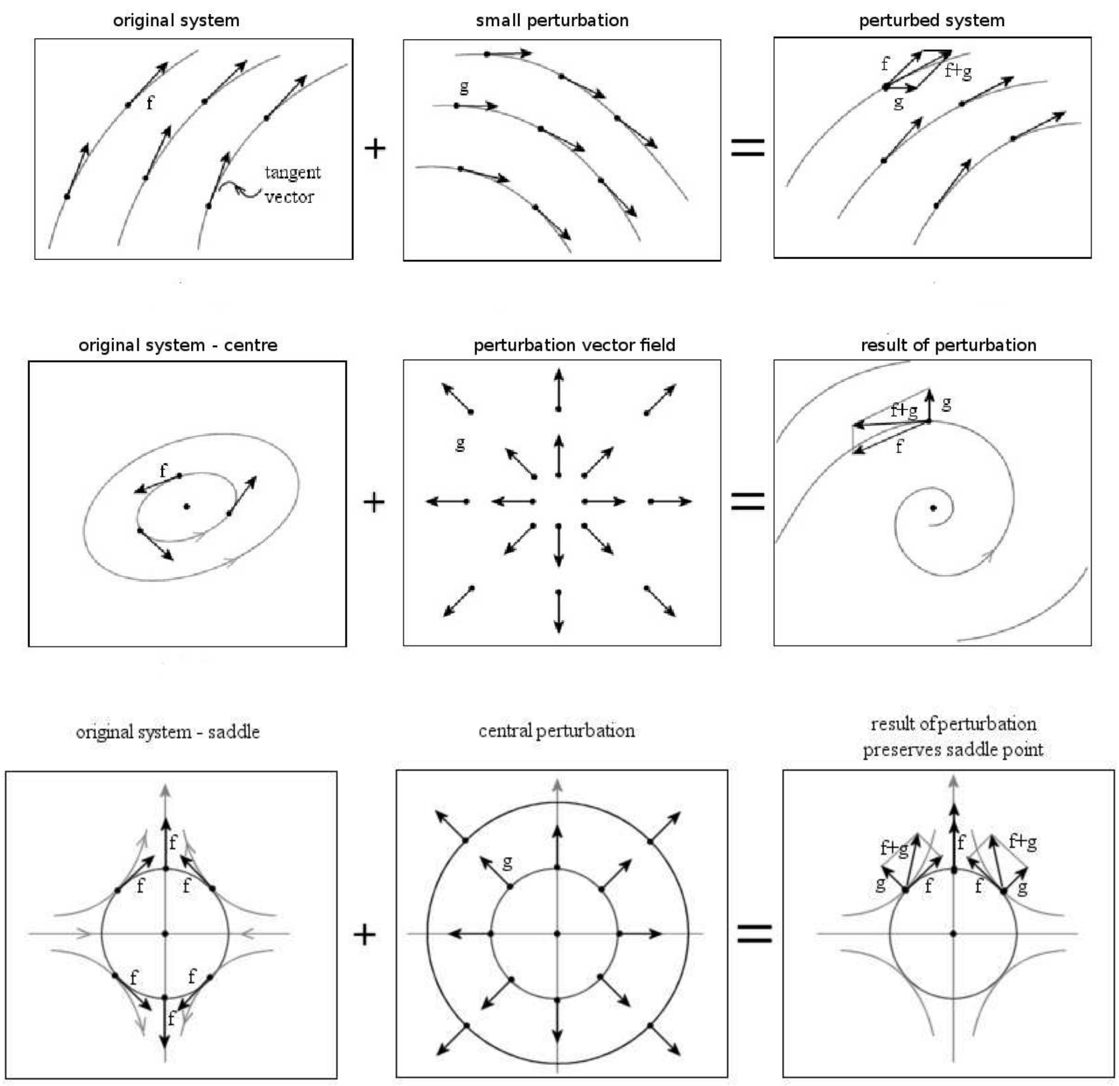


Figure 1: The schematic illustration of a) structural stability of dynamical system under perturbation; b) structural instability of a center; c) structural stability of a saddle.

is played by the cosmological radius (or redshift $z$: $1+z=\frac{a_0}{a}\equiv x^{-1}$). We can construct an effective potential, the second order acceleration equation has exactly the Newtonian form, where the role of a coordinate variable is played by the cosmological radius.

Using the notion of the structural stability introduced first by Andronov, Leontovich and Pontryagin in thirties, one can classify different models of cosmic acceleration. In will be demonstrated that models with the accelerating phase which follows the deceleration are natural and typical from the point of view of the dynamical systems theory combined with the notion of structural stability in contrast to the models with bounces. In Fig. 2 there are illustrated two cases: a) inverted single-well potential and b) more complicated form of the potential with two maxima corresponding to the saddle point and minimum corresponding to the center (structurally unstable).

Let us introduce the following definition:

**Definition 1** *If the set of all vector fields* $\boldsymbol{f}\in C^r(\mathcal{M})$ $(r\geq 1)$ *having a certain property contains an open dense subset of* $C^r(\mathcal{M})$*, then the property is called generic.*

From the physical point of view it is interesting to know whether certain subset $v$ of $C^r(\mathcal{M})$ (representing a class of cosmological accelerating models in our case) contains a dense subset because it means that this property (acceleration) is typical in $V$ (see Fig.1).

It is not difficult to establish some simple relation between the geometry of potential function and the localization of critical points and its character for the case of dynamical systems of Newtonian type:

1. The critical point of the system under consideration $\dot{x}=y$, $y=-\frac{\partial V}{\partial x}$ lies always on $x$-axis, i.e. they are representing static universe $y_0=0$, $x=x_0$;
2. The point $(x_0,0)$ is a critical point of the Newtonian system if it is a critical point of the potential function $V(x)$, i.e. $V(x)=E$ ($E$ is total energy of the system $E=\frac{y^2}{2}+V(x)$; $E=0$ for the case flat models and $E=\frac{-k}{2}$ in general);
3. If $(x_0,0)$ is a strict local maximum of $V(x)$, it is a saddle type critical point;
4. If $(x_0,0)$ is a strict local minimum of the analytic function $V(x)$, it is a center;
5. If $(x_0,0)$ is a horizontal inflection point of the $V(x)$, it is a cusp.

Therefore the geometry of potential function will determine the critical points as well as its stability. The integral of energy defines the algebraic curves in the phase space $(x,y)$ which are representing the evolution of the system with time. In any case the eigenvalues of the linearization matrix satisfy the characteristic equation $\lambda^2+\frac{\partial^2 V}{\partial x^2}|_{x=x_0}=0$.

## 3 Cosmological models as dynamical systems

The cosmology is based on the Einstein field equation which represents a very complicated system of partial nonlinear differential equation. Fortunately, the majority of main class of cosmological models from the point of view of observational data, belong to the class of the spatially homogeneous ones, for which has sense the absolute cosmological time. As a consequence, the evolution of such models can be reduced to the systems of ordinary differential equations. Hence to the cosmology could be naturally applied the methods of dynamical system theory or qualitative theory of differential equation. Among these class of models especially interesting are the cosmological models with maximally symmetric space sections, i.e. homogeneous and isotropic. They are called FRW models (Friedmann-Robertson-Walker) if source of the gravity is a perfect fluid described in terms of energy density $\varrho$ and pressure $p$, both are the functions of cosmological time $t$. The FRW dynamics is described by two basic equations:

$$\ddot{a} = -\frac{1}{6}(\varrho + 3p)a = -\frac{\partial V}{\partial a} \tag{1}$$

$$\dot{\varrho} = -3H(\varrho + p);, \tag{2}$$

where the potential $V = -\frac{1}{6}\varrho a^2$, $a$ is the scale factor and $H = d\ln a/dt$ is Hubble's function, a overdot means the differentiation with respect to the cosmological time $t$.

The first equation is a consequence of the Einstein equations for the component (1,1), (2,2), (3,3) and the energy momentum tensor $T^{\mu}_{\nu} = \mathrm{diag}\,|-\varrho, p, p, p|$. This equation is called the Raychaudhuri or acceleration equation. The second equation represents the conservation condition $T^{\mu}_{\nu;\mu} = 0$. It is very strange and unreasonable that such two simple equations satisfactorily describe the Universe evolution at the large scales. Of course there is a more general class of cosmological models called the Bianchi models which has only the symmetry of homogeneity but they do not describe the current Universe which is isotropic as indicated measurement of the cosmic microwave background (CMB)radiation.

The system of equations (1) and (2) admit the first integral called the Friedmann equation

$$\varrho - 3H^2 = 3\frac{k}{a^2}, \tag{3}$$

where $k$ is curvature constant $(0, \pm 1)$ and $\varrho$ plays the role of effective energy density.

If we consider the Lambda CDM model then

$$\varrho_{\text{eff}} = \varrho_{\text{m},0} a^{-3} + \Lambda, \tag{4}$$

i.e. energy density is a sum of dust matter (cold) and dark energy. Therefore the potential function for the flat FRW model assumes the following form:

$$V = -\frac{\varrho_{\text{eff}} a^2}{6} = (-)\{\varrho_{\text{m},0} a^{-1} + \Lambda a^2\}; \tag{5}$$

or in terms of redshift

$$V(z) = -\frac{1}{6}\{\varrho_{\text{m},0}(1+z) + \Lambda(1+z)^{-1}\}$$

Formally the curvature effects as well as the cosmological constant term can be incorporated into the effective energy density ($\varrho_k = -\frac{k}{a^2}$; $\varrho_\Lambda = \Lambda$; $p_\Lambda = -\Lambda$).

To represent the evolutional paths of cosmological models in this form is popular since Peebles' monography [38] (see also [39] and modern applications [40, 41, 42] and references therein).

The form of equation (1) suggests the possible interpretation evolutional paths of cosmological models as a motion of a fictitious particle of unit mass in a one-dimensional potential parameterized by the scale factor. Following this interpretation the Universe is accelerating in the domain of configuration space $\{a\colon a \geq 0\}$ in which the potential is a decreasing function of the scale factor. In the opposite case if potential is a growing function of $a$ the Universe is decelerating. The limit case of zero acceleration corresponds to an extremum of the potential function.

It is useful to represent evolution of the systems in terms of the dimensionless density parameter $\Omega_i \equiv \frac{\varrho_i}{3H_0^2}$, where $H_0$ is present value of Hubble's function. For this aim it is sufficient to introduce the dimensionless scale factor $x \equiv \frac{a}{a_0}$ which measures the value of $a$ in the units of the present value $a_0$ (which we choose) and reparameterize the cosmological time following rule $t \longmapsto \tau$: $dt|H_0| = d\tau$. Hence we obtain a 2-dimensional dynamical system describing the evolution of cosmological models:

$$\begin{aligned} \frac{dx}{d\tau} &= y \\ \frac{dy}{d\tau} &= (-)\frac{\partial V}{\partial x} \end{aligned} \tag{6}$$

and

$$\frac{y^2}{2} + V(x) = \frac{1}{2}\Omega_{k,0}, \quad 1 + z = x^{-1}$$

where

$$V(x) = -\frac{1}{2}\{\Omega_{\text{eff}}x^2 + \Omega_{k,0} + \Omega_{\text{Card},0}x^{m+2}\};$$

$z$ is redshift;

$$\Omega_{\text{eff}} = \Omega_{\text{m},0}x^{-3} + \Omega_{x,0}x^{-3(1+w_x)}$$

for dust matter and quintessence matter satisfying the equation of state $p_x = w_x \varrho_x$, $w_x = \text{const}$.

The form (6) of the dynamical system opens the possibility of adopting dynamical system methods in investigations of all possible evolutional scenarios for all possible initial conditions. Theoretical research in this area obviously shift from founding and analyzing particular cosmological solution to investigating a space of all admissible solutions and discovering how certain properties (like acceleration, existence of singularities for example) are "distributed" in this space. The system (6) is Hamiltonian one and adopting Hamiltonian formalism into the admissible motion seems to be natural. This gives at once insight into dynamics of accelerating Universe because our problem is similar to the problems of classical mechanics. It is achieved due to particle like description of accelerating cosmology. This cosmology identifies the unique form of the potential function $V(x)$. Different potential functions for different propositions of solving the acceleration problem contains Table 1.

# 4 Emergence of the LCDM model from the CDM model in the framework of structural stability

Before we start more accurate analysis concerning postulated emergence occurrence between the LCDM and CDM models we first put it into a more general perspective. In spite of many problems concerning theoretical weakness, the relation of emergence is becoming more and more popular. It appears to be applicable not only in biological sciences, but also in mathematics, quantum mechanics, chaos theory, physical chemistry or the philosophy of mind. The problems and controversies with formulation a coherent conception of emergence concern in principle a few areas susceptible to critics. Firstly, in discussion with reductionism and physicalism, emergence reveals weakness in the explanation of its mechanism: how do the new levels of structures emerge. We often do not know how emergence works or if the emergent theory explains at all. Secondly, there are many candidates for *the unit* of emergence. Is it a structure, information, law, property or process, event, new effect maybe? It appears that on every level of organization, where we notice existence of emergent properties, we need the new theoretical instruments. Thirdly, variety of emergences makes difficulty with elaborating of its unifying theory. It should be noticed that

Table 1: The potential function for different dark energy models

| model | potential function | independent parameters |
|---|---|---|
| Einstein – De Sitter model<br>$\Omega_{\mathrm{m},0}=1$, $\Omega_{\Lambda,0}=0$<br>$\Omega_{k,0}=0$ | $V(x)=-\frac{1}{2}\Omega_{\mathrm{m},0}x^{-1}$ | $H_0$ |
| $\Lambda$CDM model<br>$\Omega_{\mathrm{m},0}+\Omega_{\Lambda,0}=1$ | $V(x)=-\frac{1}{2}\{\Omega_{\mathrm{m},0}x^{-1}+\Omega_{\Lambda,0}x^{2}\}$ | $(\Omega_{\mathrm{m},0},\ H_0)$ |
| FRW model filled with<br>noninteracting multifluids<br>$p=w_i\varrho_i$ with dust,<br>matter and curvature | $V(x)=-\frac{1}{2}\{\Omega_{\mathrm{m},0}x^{-1}+\Omega_{k,0}+$<br>$+\sum_{i=1}^{n}\Omega_{i,0}x^{-3(1+w_i)}\}$ | $(\Omega_{\mathrm{m},0},\ H_0$<br>$\Omega_{\Lambda,0},\Omega_{i,0})$ |
| FRW quintessence model<br>with dust and dark matter $x$<br>$p_x=w_x\varrho_x$, $w_x=\mathrm{const}$ | $V(x)=-\frac{1}{2}\{\Omega_{\mathrm{m},0}x^{-1}+\Omega_{k,0}+$<br>$+\Omega_{x,0}x^{-1-3w_x}\}$<br>$w_x<-1$ phantom models | $(\Omega_{\mathrm{m},0},\ H_0,\ \Omega_{x,0})$<br>$\Omega_{k,0}=1-\Omega_{\mathrm{m},0}$<br>$-\Omega_{x,0}$ |
| FRW model with baryonic matter<br>generalized Chaplygin<br>gas [4] $p=-\frac{A}{\varrho^{\alpha}}$, $A>0$ | $V(x)=-\frac{1}{2}\{\Omega_{\mathrm{m},0}x^{-1}+\Omega_{k,0}+$<br>$+\Omega_{\mathrm{Chapl},0}(A_s+\frac{1-A_s}{x^{3(1+\alpha)}})^{\frac{1}{1+\alpha}}\}$ | $(\Omega_{\mathrm{m},0},\ H_0,\ \Omega_{\mathrm{Chapl},0})$ |

the very notion of emergence evolutes and applicated to physics shows quite different faces. We suggest that while having philosophical flavour, emergence notion should be treated with the great caution. A special attention should be paid to which or whose version of emergence we use (Broad's, Humphrey's or Kim's notion of emergence for instance) [43].

The sense of emergence notion is described usually in terms of the distinction between epistemology and ontology. It will be shown that this kind of theoretical framework is not sufficient. While describing emergence it is often useful to apply to it two important predicates: interacting or actualizing (emergence). An interacting emergence means that the new feature of the system can be explained on the basis of relation (interaction) between elements of this system. Actualization in emergence concerns particular properties existing in parts of the whole. While speaking of emergence, first association points at novelty. It should be asked in the most general way: what does the novelty consist in? Kim suggests that the novelty occurs in the two different ways: when something (property for example) is unpredictable or because of causal powers which can be detected in the relation between parts and wholes [44]. On the other side, novelty is such an unclear concept that it is obviously not good criterion for establishing the levels of empirically examined reality. Epistemology gives maybe better tools for finding appropriate description of the emergence mechanism, when we recognize the nature of the laws (biological, chemical, physical) which determine the properties of the specific level. Shortly, the quality of the language used in description of it determines something like detector of the appropriate stratum of the *universum*. The meta-theoretical framework of our paper can be expressed in a few items:

- In this paper we focus not only on the emergence existing in the description of properties (epistemological version)[3], but also on the methodological implications which concern the explanatory power of two coexisting ways of description (the CDM and LCDM models).

[3]"The description of properties at a particular level of description offers necessary but not sufficient conditions to derive the descriptions of properties at a higher level." [12, p.1757]

- We prefer using the expression *the ways (or methods) of description* instead of *the levels of description* because we are going to achieve a specific goal—quite risky one: we want to show that it is possible to speak of the relation of emergence which occurs between two models on the same level of description. There is a hierarchy of description, as suggested by Bishop and Atmanspacher, but that hierarchy has two important features: 1. it is diachronical—the LCDM model does not supervene on the CDM model; 2. it is strongly conditioned on observational data and thus determined by the current state of scientific knowledge.

- Although Popper has not elaborated clear and coherent conception of emergence [**?**], in our opinion it is possible to use his scheme of the growth of knowledge to explain that kind of emergence we are dealing with:

  $$P_1(\textit{problem}) \rightarrow TT(\textit{tentative theory}) \rightarrow EE(\textit{error ellimination}) \rightarrow P_2(\textit{different problem})$$

  Emergence occurs when the theory *collaborates with* the observational data and it concerns not only the new problems (the emergence of problems) but also the change of theory itself. Hajduk has shown that one can go beyond the philosophy of K. R. Popper and generalize this scheme as to formulate a more accurate model of theory dynamics [45]:

  $$\begin{tikzcd}
 & T_{EP} & & & T_{EP} & & & T_{EP} & & & T_{EP} & \\
BT \arrow[r] \arrow[ur] \arrow[dr] & EE \arrow[r] & P_1 \arrow[r] & TT_1 \arrow[r] \arrow[ur] \arrow[dr] & EE \arrow[r] & P_2 \arrow[r] & TT_2 \arrow[r] \arrow[ur] \arrow[dr] & EE \arrow[r] & P_1 - P_n \arrow[r] & T_1 - T_n \arrow[r] \arrow[ur] \arrow[dr] & EE & \cdots \\
 & T_{RP} & & & T_{RP} & & & T_{RP} & & & T_{RP} &
\end{tikzcd}$$

  Where $T_{EP}$ – the explanatory power of the theory (it makes the theory deals with known problems); $T_{RP}$ – the resolving power of the theory (it contributes to new problems arising); $P_1 - P_n$ – new problems with radically preformulated versions of the old ones; $T_1 - T_n$ – the new theories being applied to $P_1 - P_n$.

The dynamical system investigation of the solutions of differential equations shifts key point from founding and analyzing of individual solutions to investigating the space of all solutions for all admissible initial conditions, in the geometrical language of the phase space. Certain property (such as acceleration, singularities etc.) is believed to be realistic if it can be attributed to a large subset of models within the space of all solutions [25]. The evolutional scenarios are represented by the phase curves or by critical points, limit circles or the other limit sets. We say that two dynamical systems (or equivalently vector fields), say $f(x)$ and $g(x)$) if there is an orientation preserving homeomorphism sending integral curves of $f$ into those of $g$. Of course this equivalence relation divided space of all dynamical systems on the plane on disjoint class of abstraction. Let phase space $E = R^n$, then $\epsilon$ - perturbation $f$ is the function $g \in C^1(\mathcal{M})$ satisfying $\|f - g\|_1 < \epsilon$; where $\mathcal{M}$ is open subset of $R^n$ and $\|...\|_1$ is $C^1$ norm form the Banach space. In the introduced language it is natural to formulate an idea of structural stability. The intuition is very simple, namely $f$ is structurally stable vector field if for any vector field $f$ and $g$ are topologically equivalent. Then one can define the property of structural stability of the system.

**Definition 2** *A vector field $f \in C^1(\mathcal{M})$ is called to be structurally stable if there is an $\epsilon > 0$ such that for all $g \in C^1(\mathcal{M})$ with $\|f - g\|_1 < \epsilon$, $f$ and $g$ are topologically equivalent on open subset $R^n$ called $\mathcal{M}$.*

The 2-dimensional case is distinguished by the fact that the Peixoto theorem (1962) gave a complete characterization of structurally stable systems on any compact, two dimensional space asserts that they are generic, i.e. forms open and dense subsets in the space of all dynamical system on the plane [37].

While there are no counterpart to the Peixoto theorem in higher dimension it can be easy used to test whether such dynamical systems or cosmological origin has a structurally stable global phase portrait. In particular, a vector field on the Poincaré sphere will be structurally unstable if there are non-hyperbolic critical points at infinity on the equator of the Poincaré sphere. In the opposite case if additionally the number of critical points and limit cycles is finite $f$ is structurally stable on $S^2$.

In this section we will prove that the CDM model is structurally unstable (therefore exceptional in the space of all dynamical systems on the plane) and it transition (which we called emergence) to the Lambda CDM model means perturbation of the CDM system such that new perturbed system is structurally stable (therefore generic). Moreover the LCDM system can be treated as abstract of equivalence principle (therefore representative case) introduced in the class of accelerating cosmological models. In other words all global phase portrait equivalent to LCDM cosmological models or the potential diffeomorphic to the inverted single well potential. We assume that class of the FRW dynamical systems with the dark energy can be described in terms of the single potential function of the scale factor or redshift. If dark energy is described in terms of the coefficient of the equation of state $w_X(z) = \frac{p_X}{\varrho_X}$ then the above assumption is always satisfied.

Let us rewrite the acceleration equation (1) in the new variable $z$: $1 + z = a^{-1}$. Then we obtain

$$\ddot{z} = 2(1+z)^{-1}\dot{z}^2 + \frac{1}{6}\varrho(1+3w(z))(1+z). \tag{7}$$

The equation (7) represents a special case of a more general type of the equations

$$\ddot{z} = f(z)\dot{z}^2 + g(z). \tag{8}$$

For such a type of equations one can always eliminate the term $f(z)\dot{z}^2$ by the reparameterization of the original time variable $t$, $\dot{} \equiv \frac{d}{dt}$, namely

$$t \longmapsto \tau\colon \frac{dt}{d\tau} = h(z), \quad {}' \equiv \frac{d}{d\tau}. \tag{9}$$

For this aim it is sufficient to choose

$$h(z) = \exp(-\int^z f(z)\,dz). \tag{10}$$

Then we rewrite the equation (7) to the new form

$$z'' = g(z)\exp\{(-2)\int^z f(z)\,dz\} \equiv (-)\frac{\partial V}{\partial z}. \tag{11}$$

The equation (11) represents the evolution of the FRW dynamical system with dark energy. The potential function is given by the formula

$$V(z) = -\int^z e^{(-2)\int^z f(z)\,dz} g(z)\,dz. \tag{12}$$

In the case considered $f(z)$ and $g(z)$ are determined by (7) and the potential function reduces to the form

$$V(z) = -\frac{1}{6}\int \frac{\varrho(1+3w(z))}{(1+z)^3}\,dz, \tag{13}$$

where $\varrho$ plays the role of the effective energy density.

It is easily to check that the above formula can be exactly integrated by part if we assume that $\varrho$ satisfies the adiabatic condition (2). The final result is:

$$V(z) = -\frac{1}{6}\frac{\varrho}{(1+z)^2} \tag{14}$$

and $z^{''} = -\frac{\partial V}{\partial z}$, i.e. dynamics of the FRW model with dark energy reduces to the 2-dimensional dynamical system of a Newtonian type. The above equation has very simple interpretation as the motion of a particle of the unit mass in the potential well. During the motion of the system the total energy is preserved, i.e.

$$\frac{z^{'2}}{2} + V(z) = E = const, \tag{15}$$

where $E = -\frac{k}{2}$.

The Lagrangian of the fictitious particle which mimics the evolution of the cosmological model has the form

$$\mathcal{L} = \frac{1}{2}(z^{'})^2 - V(z),$$

where $\frac{dt}{d\tau} \equiv h(z) = (1+z)^{-4}$ or

$$\mathcal{L} = \frac{1}{2M^{-1}(z)}(\frac{dz}{dt})^2 - V(z), \tag{16}$$

where $M(z) = (1+z)^4$.

The form of Lagrangian is natural, therefore Hamiltonian assumes the following form:

$$\mathcal{H} = \frac{1}{2M(z)}p_z^2 + \tilde{V}(z), \tag{17}$$

where $p_z$ is momentum conjugated with the positional variable $z$, $R'^2 = -\frac{1}{2}\Omega_{k,0} - \tilde{V}(z) \geq 0$ is domain admissible for the motion. It is convenient to rewrite it using dimensionless variables of the density parameters instead of the energy density $\varrho_i$.

For this aim it is sufficient to reparameterize the time variable

$$t \longmapsto |H_0|\tau = \sigma, \tag{18}$$

where $H_0$ is the present value of the Hubble's function.

Finally we obtain the Hamiltonian formulation of the dynamics of the FRW model with dark energy.

$$H \longmapsto \tilde{H} = \frac{1}{2M}(\frac{dz}{|H_0|\,dt})^2 + \tilde{V}(z) = \frac{1}{2M}(\frac{dz}{d\sigma})^2 + \tilde{V}(z), \tag{19}$$

where

$$\tilde{V}(z) = (-)\frac{1}{2}\Omega_{\text{eff}}(1+z)^{-2}, \quad E = \frac{1}{2}\Omega_{k,0}.$$

In the special cases of the CDM and LCDM models the potential function is in the form

$$\tilde{V}(z) = -\frac{1}{2}\Omega_{\text{m},0}(1+z) \tag{20}$$

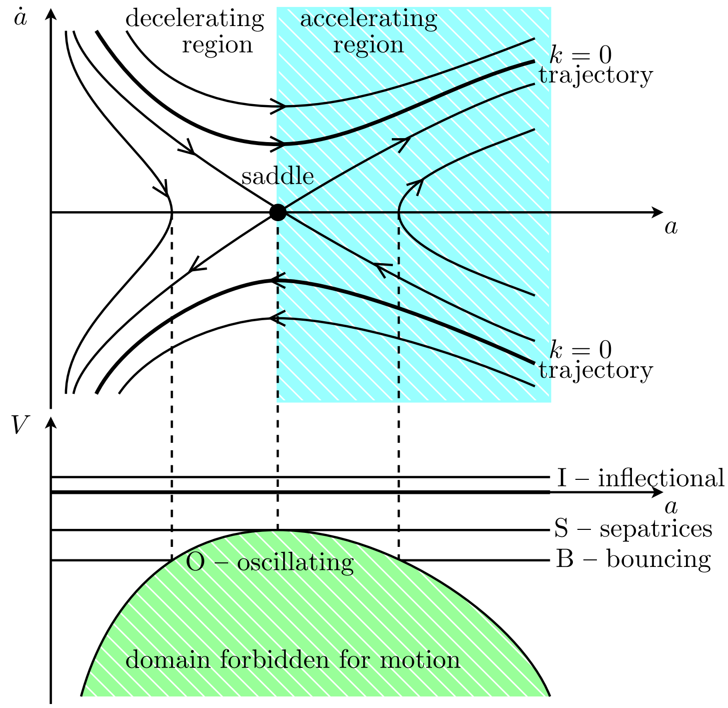


Figure 2: The phase portrait for the perturbed potential $V(x)$ on the phase plane $(x, x')$ at the finite domain. Here redshift $z = x^{-1} - 1$.

for CDM model where $\Omega_{\mathrm{m},0} = 1$ if $\Omega_{k,0} = 0$ (flat model), and

$$\tilde{V}(z) = (-)\frac{1}{2}\{\Omega_{\mathrm{m},0}(1+z) + \Omega_{\Lambda,0}(1+z)^{-2}\}, \tag{21}$$

for the LCDM model.

All density parameters are not independent and satisfy constraints conditions as a consequence of the conservation energy integral $\tilde{H} = E = -\Omega_{k,0}$

$$\sum_i \Omega_{i,0} + \Omega_{k,0} = 1, \tag{22}$$

where we apply $z = 0$ or that

$$\tilde{V}(z=0) + \frac{1}{2} = \Omega_{k,0}.$$

The potential (21) can be treated as a perturbation of potential (20) which is manifested for redshift $z < z_{\mathrm{trans}}$. The phase portrait for the LCDM model as well as its potential $V(x)$ illustrates Fig. 2. The saddle point in the phase portrait corresponds to a maximum of the potential function. The universe is decelerating for $x < x(z_{\mathrm{trans}})$ and accelerating in the domain $x > x(z_{\mathrm{trans}})$.

Let us define some class of the perturbed CDM models. Such a class can be defined in terms of the perturbed system of a Newtonian form.

**Definition 3** *By the perturbed CDM model we understand 2-dimensional dynamical system of a Newtonian form* $\ddot{a} = -\frac{\partial V}{\partial a}$ *(or* $z'' = -\frac{\partial V}{\partial z}$*) with the potential function*

$$V = V_{CDM} - \frac{1}{6}\sum_{0,\pm1,\pm2} \varrho_{i,0} a^{-i+2} = V_{CDM} + V_{pert}, \tag{23}$$

Table 2: Different perturbations of the CDM model

| i | interpretation |
|---|---|
| 0 | cosmological constant |
| 1 | 2D topological defects $p = -\frac{2}{3}\varrho$ |
| -1 | phantoms $p = -\frac{4}{3}\varrho$ |
| -2 | superphantoms $p = -\frac{5}{3}\varrho$ |
| 2 | 1D topological defects $p = -\frac{1}{3}\varrho$ |

*where* $V_{CDM} = -\frac{1}{2}\Omega_{m,0}x^{-1}$, $\Omega_{m,0} = 1$, $1 + z = x^{-1}$.

Table 2 contains physical interpretation of perturbation which corresponds to additional fluid contents. For different perturbation types considered only the first one (the cosmological constant) gives rise to the structural stable system.

It is the interesting question whether the global dynamics of the CDM model is structurally stable under a perturbation term. The global structure of dynamics (phase portraits) depends on the geometry of potential function because its localization as well as character depends on the first and the second derivatives of potential function $(\frac{\partial V}{\partial x})_{x=x_0} = 0$, $\lambda^2 + (\frac{\partial^2 V}{\partial x^2})_{x_0} = 0$ respectively, where $\lambda_{1,2}$ are eigenvalues of the linearization matrix of the system and are a solution of the characteristic equation $\lambda^2 - \operatorname{tr} A\lambda + \det A = 0$.

We reduce the dynamics to the 2-dimensional system in the form

$$\dot{x} = y$$
$$\dot{y} = -\frac{\partial V}{\partial x}$$

( or $z' = y$; $y' = -\frac{\partial V}{\partial z}$), where $\frac{y^2}{2} + V(x) = E = -\frac{1}{2}\Omega_{k,0} = \text{const}$ is the constant of energy.

From the above equation one can be seen that all critical points (right-hand sides of the system are vanishing) are situated on the axis $x$ ($y = 0$). From the characteristic equation we obtain that for the dynamical system under consideration only three types of critical points are admissible:

1. saddle if $x_0$: $(\frac{\partial V}{\partial x})_{x_0} = 0$ and $(\frac{\partial^2 V}{\partial x^2})_{x=x_0} < 0$;
2. focus if $(\frac{\partial^2 V}{\partial x^2})_{x=x_0} > 0$;
3. degenerated critical point if $(\frac{\partial^2 V}{\partial x^2})_{x=x_0} = 0$.

Therefore in the first case the eigenvalues are real of opposite signs, and in the second one they are purely imaginary. Because the center and degenerated (non-hyperbolic) critical points are structurally unstable only in the presence of single saddle point to guarantee the structural stability of the system at finite domain. The critical points $x_0$ of the perturbed system satisfy the condition

$$\Omega_{\mathrm{m},0} = \sum_{0,\pm1,\pm2} \Omega_{i,0}(2 - i)x_0^{3-i}.$$

Therefore at least there is only present such a single critical point.

Because the second derivative of the potential function is always upper convex, i.e., $(\frac{\partial^2 \tilde{V}}{\partial a^2}) < 0$ and critical point if exists is saddle type. If we consider only Lambda term in

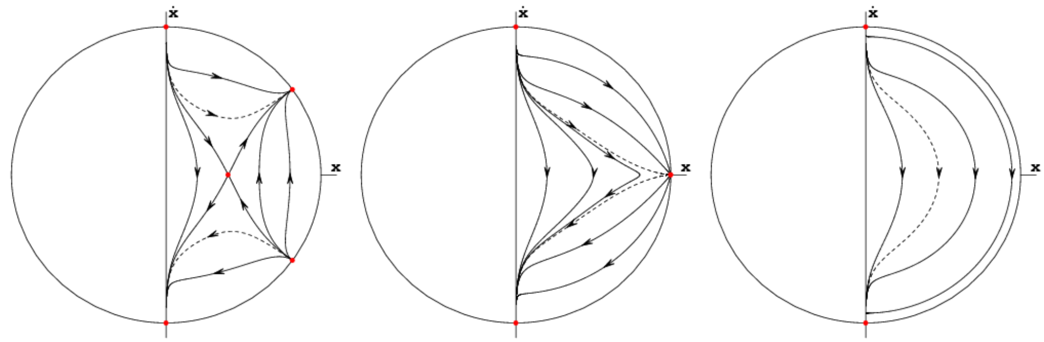


Figure 3: The phase portraits for different perturbations of the CDM model – from the left side: 1) the LCDM model with the positive cosmological constant, 2) the LCDM model with the negative cosmological constant, 3) the CDM model with the vanishing cosmological constant. Note that only the systems with the cosmological constant are structurally stable while the CDM model is unstable because of the presence of degenerated critical points at the circle at infinity (case 3). The right figure represents the Einstein static universe $(x, \dot{x}) = (\infty, 0)$. The critical point $(0, \infty)$ represents the big-bang singularity (an unstable node).

the perturbation (i.e. the LCDM model) then other terms do not change the global phase portraits of the LCDM system or all perturbed systems are topologically equivalent. The relation of global dynamics on the phase plane is the equivalence relation, therefore the LCDM model can be treated as a representative model in this class.

After the introducing the projective map covering a circle at infinity $(x, y) \mapsto (v, w)\colon v = \frac{1}{y}, w = \frac{x}{y}$ one can check that the system admits the critical point $(v_0 = 0, w_0 = 0)$ which corresponds to $\dot{x} = \infty$ and $\frac{1}{H} = 0$ (or $H = -\infty$), i.e., the big-rip singularity. This critical point is degenerate, therefore the whole system is structurally unstable. The phase portraits of the CDM model and the LCDM models with an adjoint circle at infinity $x^2 + \dot{x}^2 = \infty$ are shown in Fig. 3[4].

Note that if we consider oscillatory universes with evolution describing by the center type of critical points (not the limit cycle), then such models are nontypical from the point of view of the structural stability. On the other hand, if we consider the CDM models perturbed by Chaplygin gas then we obtain the phase portrait equivalent to LCDM.

While CDM system is structurally unstable because of the presence of non-hyperbolic critical points, the LCDM model is structurally stable. The following statement will characterize structurally stable dynamical system of Newtonian type describing perturbation of CDM models.

Let us consider more complicated $C^\infty$ class potential function $\tilde{V}(x)$ than single inverted well potential (see Fig. 6) for example with two maximum points. Thus must exist minimum critical points. But its presence means that we have a centre in the phase space, i.e. the system is structurally unstable.

**Corollary 1** *If $V$ is $C^\infty$ function of scale factor (or redshift), then there is only one differentiate type of the critical point (modulo diffeomorphism) which determines the structurally stable global phase portrait.*

[4]for comparison see [46, p. 32] where the corresponding phase portraits in the variables $(H, \varrho)$ were reproduced with and without the circle at infinity

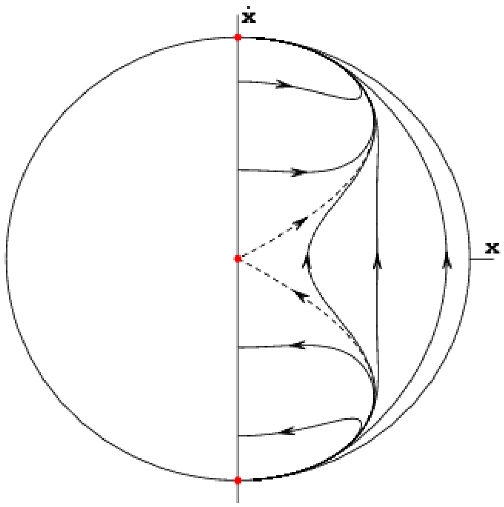


Figure 4: The phase portraits for phantom-like perturbation of the CDM model. The both systems are structurally unstable because of the presence of degenerated critical points at infinity. The critical point $(0, \infty)$ represents a big-rip singularity characteristic for the phantom cosmology. In this case the scale factor $x$ as well as its time derivative are infite at finite time. In the right figure appears an additional critical point of saddle type in the case of the negative cosmological constant.

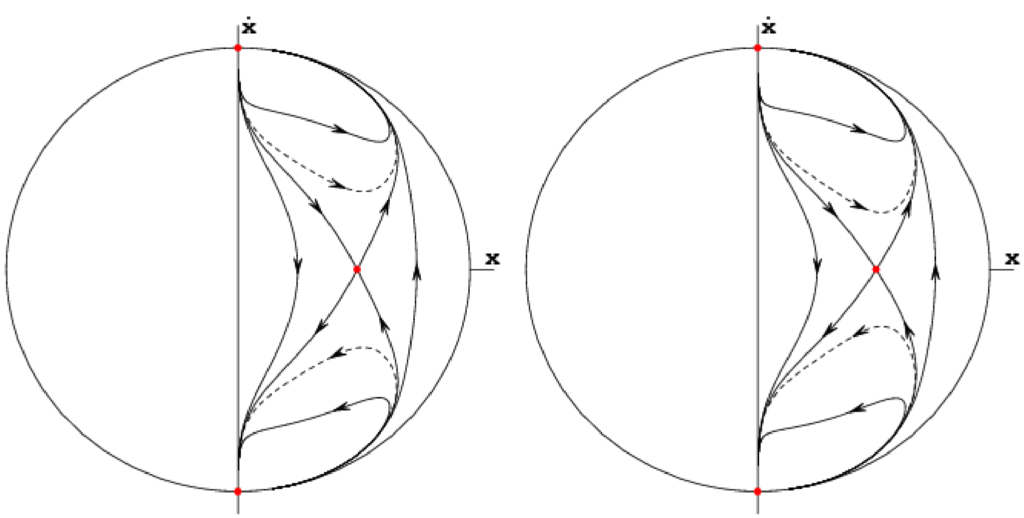


Figure 5: The phase portraits for the both LCDM models with positive (left) and negative (right) cosmological constants perturbed by the phantom contribution. They are structurally unstable because of presence of degenerate critical points at the circle at infinity. At the critical point $(0, \infty)$ the big bang singularity is glued with the big ripe one. Note that both phase portraits are topologically equivalent.

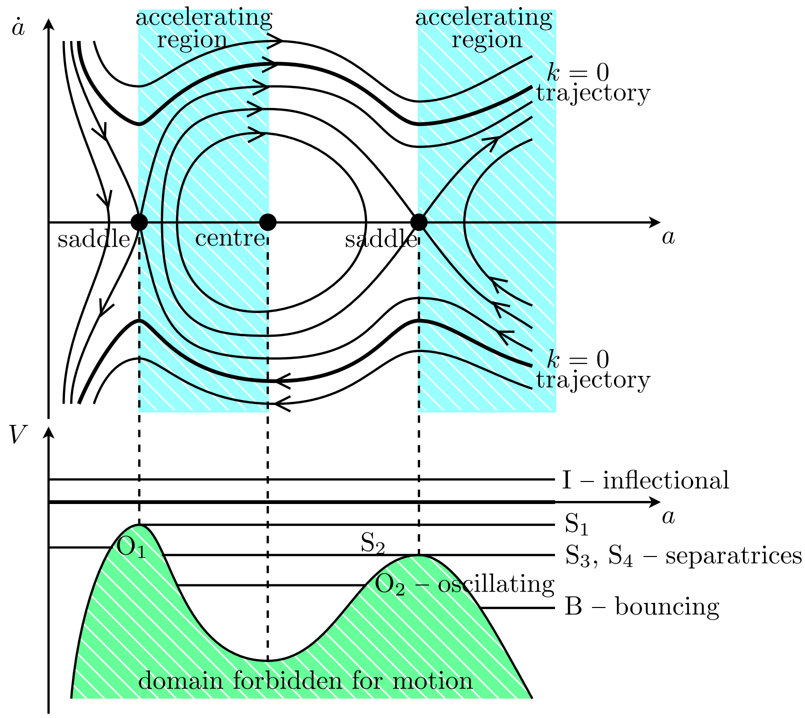


Figure 6: More complicated evolutional scenarios on $(x, x')$ plane.

This global dynamics is equivalent to the LCDM one. Finally the LCDM model is the simplest structurally stable generic perturbation of the CDM model which is nongeneric. The emergence of the LCDM model one can understand as a transition from a zero measure set of a dynamical system on the plane toward such which forms the open and the dense subsets in an ensemble of the dynamical systems on the plane – models of the deterministic processes.

# 5 Emergence of new properties of the model through the bifurcation

In our discussion it would be useful to consider a common approach to reduction in physics, so called *deductive criterion of reducibility* of Nagel [47]. In this conception reduction is a relation of derivation between upper-level and base-level theories. In upper-level theory has termed not already in the base-level one, the terms must be connected using bridge laws.

Let us consider two models which must be connected using the cosmological parameter. This parameter plays the role of control parameter in the model and we assume that it assumes zero (vanishes) in the basal model. We are looking for weakly emergent properties of the model which ca be derived (via bifurcation) from the complete knowledge of the basal model information. For this aims we use bifurcation theory, from that information about new unveiling properties of the system can be predicted, at least in principle as we change the control parameter. Then in principle we can derive the system behavior because we can perform bifurcation analysis answering the question how the structure of the phase space qualitatively changes as parameter $\Lambda$ is moved. As a result we can predict its future behavior wit complete certainty. Such a point of view seems to be very close traditional conceptions of emergence (Broad, Popper, Nagel) that focus on unpredictability properties of upper-level model even given complete the basal information.

Let us illustrate our point of view in the very simple but instructive example. The dynamics of the flat cosmological models the the R-W symmetry of space-like section, cosmological constant and without the matter (only for simplicity of presentation) is governed by very simple equation (one-dimensional system):

$$\dot{x} = -x^2 + \frac{\Lambda}{3}, \tag{24}$$

where $x = H$ is Hubble parameter which measures average rate of expansion of the Universe; $\Lambda$ is here the cosmological constant parameter; $\dot{}$ denotes differentiation with respect to the cosmological time.

Of course the above system can be simply integrated in quadratures. Calculation gives

$$\sqrt{\frac{3}{\Lambda}} Arth \sqrt{\frac{3}{\Lambda}} x = (t - t_0) \tag{25}$$

or

$$th[(t-t_0)\sqrt{\frac{3}{\Lambda}}] = \sqrt{\frac{3}{\Lambda}} x, \quad x(t) = \sqrt{\frac{3}{\Lambda}} th(\sqrt{\frac{3}{\Lambda}}(t-t_0)),$$

where $t_0$ is integration constant. The equation (24) can be also integrated for the special case of $\Lambda = 0$:

$$x(t) = \frac{1}{t - t_0}. \tag{26}$$

Note that there is no transition from the solution (25) to (26) as $\Lambda \to 0$ although such a transition exist on the level of the dynamical equation.

One can observe on this example how same small changes right hand side of the system dramatically changes its solution. As a result in this system and solution emerges new asymptotic states representing de Sitter model.

The bifurcation theory serve to clarify the emergence of new properties (sometimes unexpected) of the system without the solving this equation. Let us consider system (?) in the framework of bifurcation theory. For $\Lambda > 0$ there are two critical points $\dot{x} = 0$ at $x \pm \sqrt{\frac{\Lambda}{3}}$. From the physical point of view they are representing de Sitter model (expanding and contracting). Derivative $f(x)$ $(\dot{x} = f(x))$, $Df(x, \mu) = -2x$ and $Df(\pm\sqrt{\frac{\Lambda}{3}}, \Lambda) = \mp 2\sqrt{\frac{\Lambda}{3}}1$ and we can see that the critical point at $x = \sqrt{\frac{\Lambda}{3}}$ is stable while the critical point $x = -\sqrt{\frac{\Lambda}{3}}$ is unstable. For $\Lambda = 0$, there is only one critical point at $x = 0$ and it is a nonhypebolic critical point since $Df(0,0) = 0$; the vector field $f(x) = -x^2$ is structurally unstable; $\Lambda = 0$ is a bifurcation value. For $\Lambda < 0$ there are no critical points. The phase portraits for this differential equation are shown in Fig 7.

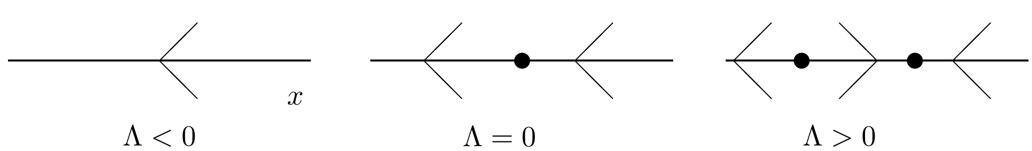


Figure 7: The phase portraits for flat FRW model and the cosmological constant of different signs

In this case we have $W^s(\sqrt{\frac{\Lambda}{3}}) = (-\sqrt{\frac{\Lambda}{3}}, \infty)$ and $W^u(-\sqrt{\frac{\Lambda}{3}}) = (-\infty, \sqrt{\frac{\Lambda}{3}})$ as a stable and unstable manifolds respectively. And for $\Lambda = 0$ the one–dimensional center manifold is given by $W^c(0) = (-\infty, \infty)$. All of the pertinent information concerning the bifurcation that takes place in this system at $\Lambda = 0$ is captured in the bifurcation diagram shown in Fig(**). The curve $\frac{\Lambda}{3} - x^2 = 0$ determines the position of the critical points of the system, a solid curve is used to indicate a family of stable critical points while a dashed curve is used to indicate a family of unstable critical points. This type of bifurcation is called a saddle-mode bifurcation.

The system under consideration constitutes only example of dynamical system analysis of the system cosmological origin but there are many other system with some parameter which shows hidden and unexpected properties as parameter varies. Let us remember some of them. In the problem of the motion star around the elliptic galaxy appears Henon, Heiles [48] hamiltonian system. This system possesses the energy first integral $E$ and if $E > E_{crit}$ then transition to the chaotic behavior appears. Another example of bifurcation and emergence of the cyclic behavior in the system of the limit cycle type offers famous van der Pol equation $\ddot{x} + \mu(x^2 - 1)\dot{x} + x = 0$. For $\mu = 0$ the system is of harmonic oscillator type and for $\mu > 0$, van der Pol's equation has a unique limit cycle and it is stable [49]. The limit cycle is representing closed trajectory in the phase space which attracts all trajectories from neighborhood.

In this case $\epsilon = 0$ is bifurcation value parameter and limit cycle behavior is upper-level emergent property. For the interesting discussion on emergence, basal and upper-level models and reducibility see [50].

Also interesting experiences of emergence new type of dynamical behavior give us Hopf bifurcation phenomena [49, s. 341]. This bifurcation can occur in the system with parameter $\dot{x} = f(x, \mu)$ at a nonhyperbolic equilibrium point $x_0$ when the matrix $Df(x_0, \mu_0)$ had a simple pair of pure imaginary eigenvalue and no other eigenvalues with zero real point. In the generic case Hopf bifurcation occurs where a periodic orbit is created as the stability of equilibrium point $x_\mu$ changes. This type of behavior plays important role in the description route to turbulence scenario. It would be worthy to mention the important role of Hopf bifurcation in Rulle-Takens scenario of route to deterministic chaos. The concept of turbulence war originally introduced by Landau in 1944 and later revised by Ruelle and Takens in 1941 [51]. According to Landau, turbulence is reached at the end of an indefinite superposition of oscillatory bifurcations, each bringing its unveiling phase into dynamics of the system. In Ruelle-Takens scenario infinite number of periodic behavior is not required when nonlinearities are acting. They argue that turbulence should be treated as a stochastic regime of deterministic chaos at which we have long term unpredictability due to property of sensitive dependance on initial condition. This stage is reached only after a finite and small number of bifurcation. For recent philosophical discussion of significance of chaos see [52].

In conventional methodology of deriving Einstein equation one derives the equation of motion from the Lagrangian which is sum of $\mathcal{L}_{grav}$ - lagrangian for gravity & $\mathcal{L}_{matt} = \mathcal{L}_{matt}(g, \phi)$ - lagrangian for matter source which we assume that depends on both metric $g$ and the scalar field $\phi$. Therefore there are two different ways of introducing the cosmological constant. In the first approach we put them into the gravitational lagrangian, i.e. $\mathcal{L}_{grav} = (2\kappa)^{-1}(R - 2\Lambda_g)$, where $\Lambda_g$ is a parameter in the (low energy effective) action just like the Newtonian gravitational constant $\kappa$. The second route is by shifting the matter lagrangian $\mathcal{L}_{matt} \to \mathcal{L}_{matt} - 2\lambda_m$. Therefor a shift is clearly equivalent to adding cosmological constant $2\kappa\lambda_m$ to the $\mathcal{L}_{grav}$ [53].

The symmetry $\mathcal{L} \to \mathcal{L}_{matt} - 2\lambda_m$ is a symmetry of matter sector. The matter equation of motion do not care about $\lambda_m$. In the conventional approach gravity breaks this symmetry. This is the root case of the so-called cosmological constant problem. As long as gravitational field equation are in the form $E_{ab} = \kappa T_{ab}$, where $E_{ab}$ is some geometrical quantity ($G_{ab}$ in G.R.) the theory cannot be invariant under the shifts of the form $T^a_b \to T^a_b + \rho\delta^a_b$. Since such shift are allowed by the matter sector it is very difficult to imagine solution to cosmological constant within the conventional approach to gravity [54, s. 11]. If the metric represents the gravitational degree of freedom that is varied in the action and we demand full general covariance, we cannot avoid $\mathcal{L}_{matt}\sqrt{-g}$ coupling and cant obtain of the equation of motion which are invariant under the shift $T_{ab} \to T_{ab} + \Lambda g_{ab}$. Clearly a new dramatically different approach to gravity is required.

## 6 Conclusion

We always in the mathematical modeling of physical processes try to convey the features of typical, garden – variety, dynamical systems. In mathematics the exceptional cases are more complicated and numerous, and they interrupt the physical discussion. Moreover dynamicists shared an opinion that such exceptional systems not arise very often because they are atypical. In the history of mathematical dynamics we observe how we have searched for generic properties. We would like to distinguish a class of phase portraits that are far simpler than the arbitrary ones. This programme was achieved for dynamical systems on the plane by Peixoto due to the conception of structural stability introduced by Andronov and Leontovich in 1934. The criteria for structural stability rely upon two supplementary notions: a perturbation of the phase portraits (or vector field) and the topological equivalence (homeomorphism of the state phase). A phase portrait has the property of structural stability if all sufficiently small perturbations of it have equivalent phase portraits. For example if we consider a center type of critical points then the addition of perturbation pointing outward results in a point repellor which is not topologically equivalent to the center. This is a primary example of a structurally unstable system. In the opposite case saddle type of critical point is structurally stable and the phase portrait doesn't change under small perturbation.

In this paper we define the class of FRW cosmological models filled by dark energy as a two-dimensional dynamical systems of a Newtonian type. They are characterized through the single smooth effective potential function of the scale factor or redshift. Among these class of models we distinguish typical (generic) and exceptional (nongeneric) cases with the help of structural stability notion and the Peixoto theorem. We find that the LCDM model in opposition to the CDM model is structurally stable. We demonstrate that this model represents a typical structurally stable perturbation of CDM one. Therefore the transition from the CDM model of the Universe toward the LCDM one which includes the effects of the cosmological constant can be understood as an emergence of the model from the exceptional case to the generic one. This case represents a generic model in this sense that small changes of its right-hand sides do not change the global phase portraits. In the terms of the potential, the second order differential equation one can classify different models of cosmic acceleration. It is shown that models with the accelerating phase (which follows the deceleration) are natural and typical from the point of view of the dynamical systems theory combined with the notion of structural stability.

It is interesting that the new class of Lambda perturbated solutions does not reduce to the CDM model solutions (which reveals their new quality) although the corresponding equation reduces to the CDM one after taken limit $\Omega_\Lambda \to 0$. The small value of Lambda parameter dramatically changes its asymptotic states (de Sitter asymptotic is emerged).

The Universe is accelerating for some value of redshift transition $z_{trans} \simeq 0.6$ and this phase of acceleration is followed by the deceleration phase dominated by matter. One can say that the LCDM model is emerging from the CDM model as the Universe evolves. This very simple two phases model of past evolution of the Universe give rise to its present acceleration detected by distant supernovae. Therefore the simplicity and genericity are the best guides to understanding of our Universe and its acceleration. More complicated evolutional scenarios are exceptional in the space of all models with a 2-dimensional phase space.

There are many different theoretical possibilities of explaining accelerating universe in terms of dark energy (substantial approach) or using modification of gravity (nonsubstantial approach). Among all candidates the LCDM model is favored by Bayesian selection methods [55]. These methods indicate the best model in respect to admissible data. One can ask why the LCDM model is the best one. Our answer is that LCDM possesses a property of simplicity and in the same time flexibility with respect to the data. The latter can be interpreted in the tools of the structural stability notion.

The observations indicates that we live in expanding Universe with current accelerations. It seems that this acceleration phase proceeded the deceleration phase. Provided that we assume that there was no other qualitative dynamical changes in whole evolution of the universe (at early as well as late time) the LCDM model is sufficiently complex to explain such a simple evolution of the Universe. No simpler neither the more complex model can be better description of the Universe dynamics. The future evolution of our universe is eternal expansion with the accelerating phase according to the LCDM scenario. Other possible futures given by other models are unjustified because of the structural instability. Such futures are highly improbable because they require a very special fine-tuned model to the reality.

It seems that there is possibility of an ideal description of the physical reality in such a way that our model is no more a model but described reality itself. In this case the structural stability or instability does not matter. But when as in cosmology we have a bunch of models which very roughly describe the universe evolution (the effective theories) they should accommodate the reality inside the error margin generated by the perturbation. But this feature is possessed by the structural stable models only. This is an argument in favour of dealing with structural stable models in cosmology. We have found the only structural stable two-phase model of universe dynamics with a deceleration and then acceleration phase is the LCDM model.

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